



Design and Analysis of Algorithms

SECOND EDITION |

S. Sridhar

Professor

Department of Information Science and Technology
College of Engineering, Guindy Campus
Anna University, Chennai

OXFORD
UNIVERSITY PRESS

Detailed Contents

Features of the Book iv

Preface vi

1 Introduction to Algorithms	1	3.4 Best-, Worst-, and Average-case Complexity	55
1.1 Introduction	1	3.5 Rate of Growth	56
1.2 Need for Algorithmic Thinking	1	3.5.1 <i>Measuring Larger Inputs</i>	56
1.3 Overview of Problems	3	3.6 Asymptotic Analysis	59
<i>1.3.1 Computational Problems, Instance and Size</i>	3	3.6.1 <i>Asymptotic Notations</i>	60
1.4 Overview of Algorithms	4	3.6.2 <i>Limits and Asymptotic Notations</i>	64
1.5 Need for Algorithm Efficiency	6	3.6.3 <i>Asymptotic Rules</i>	68
1.6 Fundamental Stages of Problem Solving	7	3.6.4 <i>Asymptotic Complexity Classes</i>	69
<i>1.6.1 Understanding the Problem</i>	7	3.7 Empirical Analysis and Algorithm Visualization	70
<i>1.6.2 Planning an Algorithm</i>	7	3.7.1 <i>Empirical Analysis</i>	70
<i>1.6.3 Designing an Algorithm</i>	8		
<i>1.6.4 Validating and Verifying an Algorithm</i>	9		
<i>1.6.5 Analysing an Algorithm</i>	9		
<i>1.6.6 Implementing an Algorithm and Performing Empirical Analysis</i>	11	4 Mathematical Analysis of Recursive Algorithms	76
<i>1.6.7 Post (or Postmortem) Analysis</i>	11	4.1 Introduction to Recurrence Equations	76
1.7 Classification of Algorithms	11	4.1.1 <i>Linear Recurrences</i>	77
<i>1.7.1 Based on Implementation</i>	11	4.1.2 <i>Non-linear Recurrences</i>	79
<i>1.7.2 Based on Design</i>	13	4.2 Formulation of Recurrence Equations	80
<i>1.7.3 Based on Area of Specialization</i>	13	4.3 Techniques for Solving Recurrence Equations	82
<i>1.7.4 Based on Tractability</i>	13	4.3.1 <i>Guess-and-verify Method</i>	82
		4.3.2 <i>Substitution Method</i>	84
		4.3.3 <i>Recurrence-tree Method</i>	87
		4.3.4 <i>Difference Method</i>	90
2 Basics of Algorithm Writing	17	4.4 Solving Recurrence Equations Using Polynomial Reduction	91
2.1 Computational Thinking	17	4.4.1 <i>Solving Homogeneous Equations</i>	91
<i>2.1.1 Decomposition</i>	18	4.4.2 <i>Solving Non-homogeneous Equations</i>	95
<i>2.1.2 Pattern Matching</i>	18	4.5 Divide-and-conquer Recurrences	95
<i>2.1.3 Abstraction</i>	18	4.5.1 <i>Master Theorem</i>	96
<i>2.1.4 Algorithm Design</i>	18	4.6 Transformations	101
2.2 Design Approaches	19	4.6.1 <i>Domain Transformation</i>	102
<i>2.2.1 Bottom-up Approach</i>	20		
2.3 Algorithm Specifications	21	5 Data Structures—I	107
<i>2.3.1 Guidelines for Writing Algorithms</i>	22	5.1 Data Structures and Algorithms	107
2.4 Iterative Algorithms	26	5.2 Lists	108
2.5 Recursive Algorithms	33	5.2.1 <i>Linear Lists</i>	108
2.6 Algorithm Correctness	39	5.2.2 <i>Linked Lists</i>	111
3 Basics of Algorithm Analysis	44	5.3 Stacks	115
3.1 Basics of Algorithm Complexity	44	5.3.1 <i>Representation and Operations on Stacks</i>	115
3.2 Introduction to Time Complexity	45	5.4 Queues	117
3.3 Analysis of Iterative Algorithms	47	5.4.1 <i>Queue Representation</i>	117
<i>3.3.1 Measuring Input Size</i>	47		
<i>3.3.2 Measuring Running Time</i>	48		

5.5 Trees	119	8.1.2 Advantages and Disadvantages of Divide-and-conquer Paradigm	203
5.5.1 <i>Tree Terminology</i>	119		
5.5.2 <i>Classification of Trees</i>	121		
5.5.3 <i>Binary Tree Representation</i>	122		
5.5.4 <i>Binary Tree Operations</i>	123		
5.6 Graphs	127	8.2 Merge Sort	204
5.6.1 <i>Terminologys and Types of Graphs</i>	127		
5.6.2 <i>Graph Representation</i>	130		
5.7 Elementary Graph Algorithms	131	8.3 Quicksort	208
5.7.1 <i>Graph Traversal Algorithms</i>	131	8.3.1 <i>Partitioning Algorithms</i>	209
5.7.2 <i>Cycle Detection</i>	134	8.3.2 <i>Variants of Quicksort</i>	213
5.7.3 <i>Connected Components</i>	137	8.4 Multiplication of Long Integers	214
5.7.4 <i>Strongly Connected Components</i>	138	8.4.1 <i>Divide and Conquer Approach</i>	215
5.7.5 <i>Biconnected Components</i>	139	8.5 Strassen Matrix Multiplication	217
5.7.6 <i>Spanning Tree and Minimum-cost Spanning Tree</i>	141	8.6 Tiling Problem	221
6 Data Structures—II	147	8.7 Closest-pair Problem	223
6.1 Introduction to Binary Search Tree	147	8.8 Convex Hull	225
6.2 AVL Trees	152	8.8.1 <i>Quickhull</i>	225
6.3 Binary Heaps	156	8.8.2 <i>Merge Hull</i>	226
6.4 Binomial Heaps	163	8.9 Fourier Transform for Polynomial Multiplication	227
6.5 Fibonacci Heap	168	8.9.1 <i>Improvement of Polynomial Multiplication using Value Representation</i>	228
6.6 Disjoint Sets	170	8.9.2 <i>Divide and Conquer Approach and Fourier Transform</i>	228
6.7 Amortized Analysis	173	8.9.3 <i>Fast Discrete Fourier Transform (FFT) using Divide and Conquer Approach</i>	231
6.7.1 <i>Aggregate Method</i>	174	8.9.4 <i>Polynomial Multiplication</i>	232
6.7.2 <i>Accounting Method</i>	174		
6.7.3 <i>Potential Method</i>	175		
7 Brute Force Approaches	179	9 Decrease-and-conquer Approach	237
7.1 Introduction	179	9.1 Introduction	237
7.1.1 <i>Advantages and Disadvantages of Brute Force Method</i>	180	9.2 Decrease by Constant Method	238
7.2 Linear Search	180	9.2.1 <i>Insertion Sort</i>	239
7.2.1 <i>Analysis of Recursion Programs</i>	181	9.2.2 <i>Topological Sort</i>	242
7.3 Sorting Problem	182	9.2.3 <i>Generating Permutations</i>	246
7.3.1 <i>Bubble Sort</i>	183	9.2.4 <i>Generating Subsets</i>	248
7.3.2 <i>Selection Sort</i>	185	9.3 Decrease by Constant Factor Method	249
7.4 Computational Geometry Problems	187	9.3.1 <i>Binary Search</i>	249
7.4.1 <i>Closest-pair Problem</i>	187	9.3.2 <i>Fake Coin Detection</i>	251
7.4.2 <i>Convex Hull Problem</i>	189	9.3.3 <i>Russian Peasant Multiplication Problem</i>	252
7.5 Exhaustive Searching	191	9.4 Decrease by Variable Factor Method	253
7.5.1 <i>15-puzzle Problem</i>	191	9.4.1 <i>Interpolation Search</i>	253
7.5.2 <i>8-queen Problem</i>	193	9.4.2 <i>Selection and Ordered Statistics</i>	254
7.5.3 <i>Magic Squares</i>	193	9.4.3 <i>Finding Median</i>	257
7.5.4 <i>Container Loading Problem</i>	195	9.4.4 <i>Selection (Deterministic and Random Selection)</i>	257
7.5.5 <i>Knapsack Problem</i>	195	9.4.5 <i>Finding Median using Median of Medians</i>	259
7.5.6 <i>Assignment Problem</i>	197		
8 Divide-and-conquer Approach	202	10 Time–Space Tradeoffs	264
8.1 Introduction	202	10.1 Introduction to Time-Space Tradeoffs	264
8.1.1 <i>Recurrence Equation for Divide and Conquer</i>	203	10.2 Counting Sort	264
		10.3 Bucket Sort	268
		10.4 Radix Sort	270
		10.5 Shell Sort	271
		10.6 Hashing and Hash Tables	273
		10.6.1 <i>Introduction to Hash Functions</i>	273
		10.6.2 <i>Hash Functions Construction</i>	273

10.6.3 Hash Table Operations	275		
10.6.4 Collision Resolution Techniques	276		
10.7 Introduction to M-ary Trees	278		
10.7.1 B-Tree	279		
10.7.2 B+ Trees	284		
11 Greedy Algorithms	288		
11.1 Introduction to the Greedy Approach	288		
11.2 Components of Greedy Algorithms	289		
11.2.1 Properties of Greedy Algorithms	290		
11.3 Coin Change Problem	291		
11.4 Scheduling Problems	292		
11.4.1 Shortest Job First Scheduling	292		
11.4.2 Scheduling with Deadline	294		
11.4.3 Activity Selection Problem	296		
11.5 Knapsack Problem	298		
11.6 Optimal Storage of Tapes	301		
11.7 Optimal Merge Problem	302		
11.7.1 Evaluation of Optimal Tree of Merging	303		
11.8 Huffman Coding	304		
11.8.1 Efficiency of Huffman Code Using Weighted Tree	306		
11.9 Tree Vertex Splitting Problem	308		
11.10 Minimum Spanning Trees	310		
11.10.1 Kruskal's Algorithm	310		
11.10.2 Prim's Algorithm	315		
11.10.3 Single-Source Shortest-Path Problems	319		
12 Transform-and-conquer Approach	328		
12.1 Introduction to Transform and Conquer	328		
12.2 Introduction to Instance Simplification	329		
12.3 Gaussian Elimination and Matrix Operations	330		
12.4 Applications of Gaussian Elimination	336		
12.4.1 LU Decomposition	337		
12.4.2 Solving a set of Linear Equations using LU Decomposition	340		
12.4.3 LUP Decomposition	342		
12.4.4 Solving Linear Equations using LU Decomposition	344		
12.4.5 Finding Matrix Inverse using Gaussian Elimination	344		
12.4.6 Finding Determinant using LU Decomposition	346		
12.5 Change of Representation and Heap Sort	348		
12.6 Polynomial Evaluation Using Horner's Method	351		
12.7 Binary Exponentiation	353		
12.7.1 Left-to-right Binary Exponentiation	355		
12.7.2 Right-to-left Binary Exponentiation	356		
12.8 Problem Reduction	357		
13 Dynamic Programming	361		
13.1 Basics of Dynamic Programming	361		
13.2 Components of Dynamic Programming	363		
13.3 Fibonacci Problem	365		
13.3.1 Bottom-up Approach	365		
13.3.2 Top-down Approach and Memoization	366		
13.4 Computing Binomial Coefficients	367		
13.5 Multistage Graph Problem	370		
13.5.1 Forward Computation Procedure	370		
13.5.2 Backward Computation Procedure	374		
13.6 Transitive Closure and Warshall Algorithm	376		
13.7 Floyd-Warshall All-Pairs Shortest-Path Algorithm	379		
13.8 Bellman-Ford Algorithm	382		
13.9 Travelling Salesperson Problem	385		
13.10 Chain Matrix Multiplication	387		
13.11 Knapsack Problem	394		
13.12 Optimal Binary Search Trees	397		
13.12.1 Dynamic Programming Approach for Constructing Optimal BSTs	398		
13.13 Flow-Shop Scheduling Problem	402		
13.13.1 Single-machine Sequencing Problem	402		
13.13.2 Two-machine Sequencing Problem	403		
14 Backtracking	410		
14.1 Introduction	410		
14.2 Basics of Backtracking	411		
14.2.1 Generation of State-space Trees	412		
14.2.2 Searching State-space Trees	413		
14.2.3 Implementation of Backtracking Algorithm	415		
14.3 N-Queen Problem	416		
14.4 Sum of Subsets Problem	418		
14.5 Vertex Colouring Problem	421		
14.6 Hamiltonian Circuit Problem	423		
14.7 Generating Permutation	427		
14.8 Graham Scan	428		
15 Branch-and-bound Technique	434		
15.1 Introduction	434		
15.2 Branch and Bound Technique	435		
15.3 Search Techniques for Branch-and-Bound Technique	436		
15.3.1 BFS Using Branch and Bound Algorithm—FIFOB	437		
15.3.2 LIFO with Branch and Bound	438		
15.3.3 Least Cost with Branch and Bound	438		
15.4 15-puzzle Game	439		
15.5 Assignment Problem	442		
15.6 Traveling Salesperson Problem	447		
15.7 Knapsack Problem	449		

16 String Algorithms	457	18.5 Complexity Classes	521
16.1 Introduction to String Processing	457	18.5.1 Class P	522
16.2 Basic String Algorithms	458	18.5.2 Class NP	523
16.2.1 Length of Strings	458	18.5.3 NP-hard Problems	523
16.2.2 Concatenation of Two Strings	459	18.6 Theory of NP-Complete Problems	524
16.2.3 Finding Substrings	459	18.7 Reductions	525
16.3 Longest Common Subsequences	460	18.7.1 Turing Reduction	525
16.3.1 Brute Force Method	460	18.7.2 Karp Reduction	526
16.3.2 Dynamic Programming	461	18.8 Proof of NP-Completeness	527
16.4 String Edit Distance Using Dynamic Programming	465	18.9 Example Problems for Proving NP-Completeness	528
16.5 Naïve String-Matching Algorithm	468	18.9.1 SAT is NP-complete	528
16.6 Pattern Matching Using Finite Automata	469	18.9.2 Problem 3-CNF-SAT is NP-complete	529
16.7 Rabin–Karp Algorithm	471	18.9.3 Clique Decision Problem is NP-complete	530
16.7.1 Stage 1	471	18.9.4 Sum of Subsets (from 3-CNF-SAT)	531
16.7.2 Stage 2	472		
16.8 Knuth–Morris–Pratt Algorithm	474		
16.8.1 Stage 1	475	19 Randomized Algorithms	535
16.8.2 Stage 2	476	19.1 Introduction to Randomized Algorithms	535
16.9 Boyer–Moore String Matching Algorithm	478	19.2 Generation of Random Numbers	537
16.9.1 Bad-character Shift Rule	478	19.3 Hiring Problem	539
16.9.2 Good-suffix Shift Rule	478	19.3.1 Probabilistic Analysis of Deterministic Hiring Algorithm	539
16.10 Boyar-Moore- Harspool Algorithm	480	19.4 Primality Testing Algorithm	542
16.10.1 Stage 1	480	19.5 Comparing Two Strings using Randomization Algorithm	543
16.10.2 Stage 2	482	19.6 Randomized Quicksort	545
17 Iterative Improvement and Linear Programming	485		
17.1 Introduction to Iterative Improvement	485	20 Approximation Algorithms	549
17.2 Linear Programming	485	20.1 Introduction to Approximation Algorithm	549
17.2.1 Advantages and Limitations of Linear Programming	486	20.2 Goodness Factor	550
17.3 Formulation of Lpps	486	20.3 Design approaches for Approximation Algorithms	551
17.4 Graphical Method for Solving LPP	489	20.4 Heuristic Approaches for Travelling Salesperson Problem	552
17.5 Simplex Method	493	20.4.1 Nearest Neighbor	552
17.6 Minimization Problems	498	20.4.2 Multi-Fragment Heuristic Method	554
17.7 Duality	501	20.4.3 Twice-the-path Heuristic	555
17.8 Max-Flow Problem	503	20.4.4 Christofides Algorithm	556
17.8.1 Ford–Fulkerson Algorithm	504	20.5 Greedy Approximation Algorithms	558
17.8.2 Max-flow and Min-cut	506	20.5.1 0/1 Knapsack Problem	558
17.9 Bipartite Matching Problem	507	20.5.2 Continuous Knapsack Problem	559
17.9.1 Bipartite Graph	507	20.5.3 An approximation scheme for 0/1 Knapsack problem	559
17.10 Stable Marriage Problem	510	20.5.4 Vertex Cover Problem	559
		20.5.5 Set Cover Problem	561
18 Basics of Computational Complexity	516	20.6 Designing Approximation Algorithms Using Dynamic Programming	563
18.1 Introduction to Computational Complexity	516	20.6.1 Knapsack Problem	563
18.2 Upper and Lower Bound Theory	517	20.6.2 Subset Sum Problem	564
18.2.1 Methods of Finding Lower Bounds	518		
18.3 Decision Problems	519		
18.4 Turing Machines	520		
18.4.1 Languages and Turing Machine	521		

21 Parallel Algorithms	570	21.5 Simple Parallel Algorithms	578
21.1 Introduction to Parallel Processing	570	21.5.1 <i>Prefix Computation</i>	578
21.2 Classification of Parallel Systems	570	21.5.2 <i>List Ranking</i>	580
21.2.1 <i>Flynn Classification</i>	571	21.5.3 <i>Euler Tour</i>	581
21.2.2 <i>Address-space-based (or Memory Mechanism-based) Classification</i>	572	21.6 Parallel Searching and Parallel Sorting	582
21.2.3 <i>Classification Based on Interconnection Networks</i>	573	21.6.1 <i>Parallel Searching</i>	582
21.3 Introduction to PRAM Model	574	21.6.2 <i>Odd–Even Swap Sort</i>	583
21.4 Parallel Algorithm Specifications and Analysis	575	21.7 Additional Parallel Algorithms	584
21.4.1 <i>Parallel Algorithm Analysis</i>	577	21.7.1 <i>Parallel Matrix Multiplication</i>	584
		21.7.2 <i>Parallel Graph Algorithms</i>	587

Appendix A—Mathematical Basics 591

Appendix B—Proof Techniques 603

Appendix C—Python and Anaconda 612

Appendix D—Python Packages 622

Appendix E—Laboratory Manual 637

Bibliography 675

Index 677