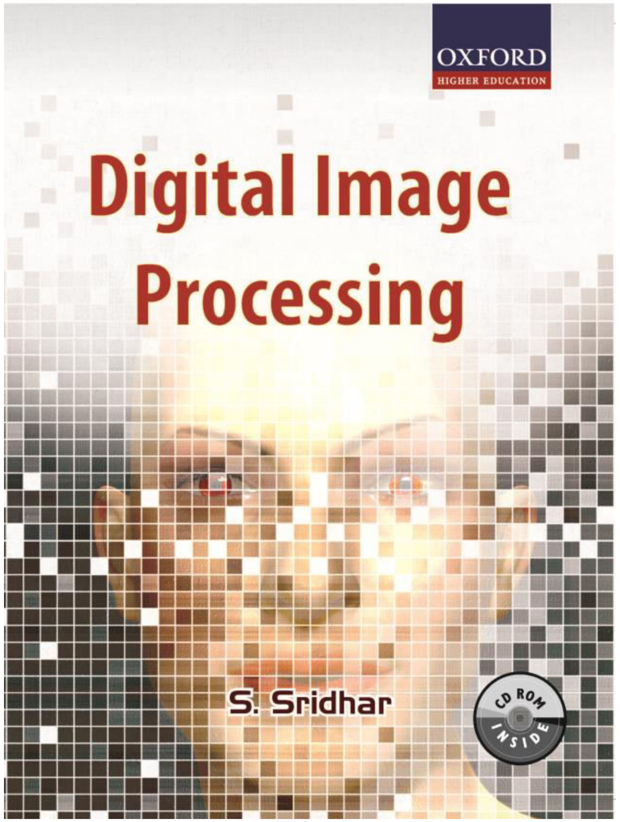

Digital Image Processing

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Chapter 6
Image Restoration

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Image Restoration

Image restoration is the process of retrieving an original image from a degraded image.

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Types of Image Degradations

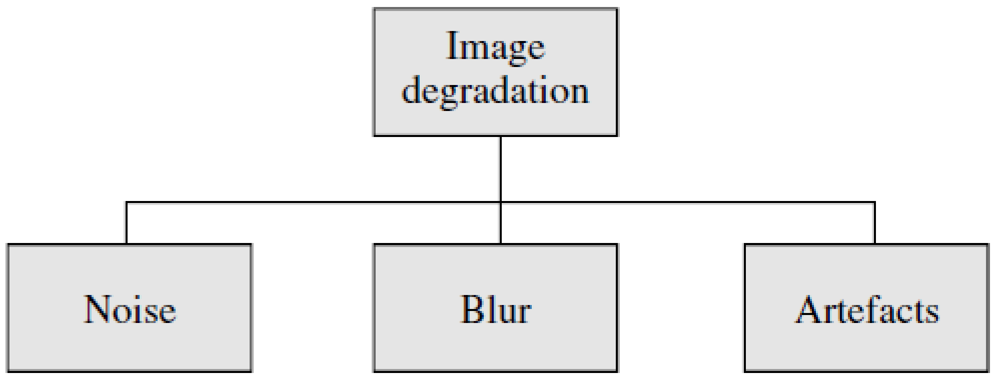

Fig. 6.1 Types of degradations

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Types of Image Degradations

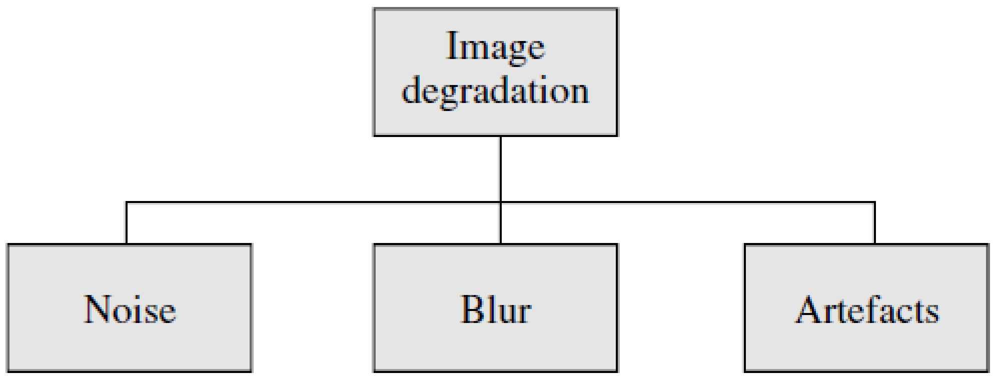

Fig. 6.1 Types of degradations

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Degradation - Noise

(a) (b)

Fig. 6.2 Effect of noise (a) Original image (b) Image with Gaussian noise

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Degradation - Blur

(a) (b) (c) (d)

Fig. 6.3 Effect of blur (a) Original image (b) Image with Gaussian blur (c) Image with motion blur (d) Image with lens-out-of-focus blur

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Degradation – Artefacts

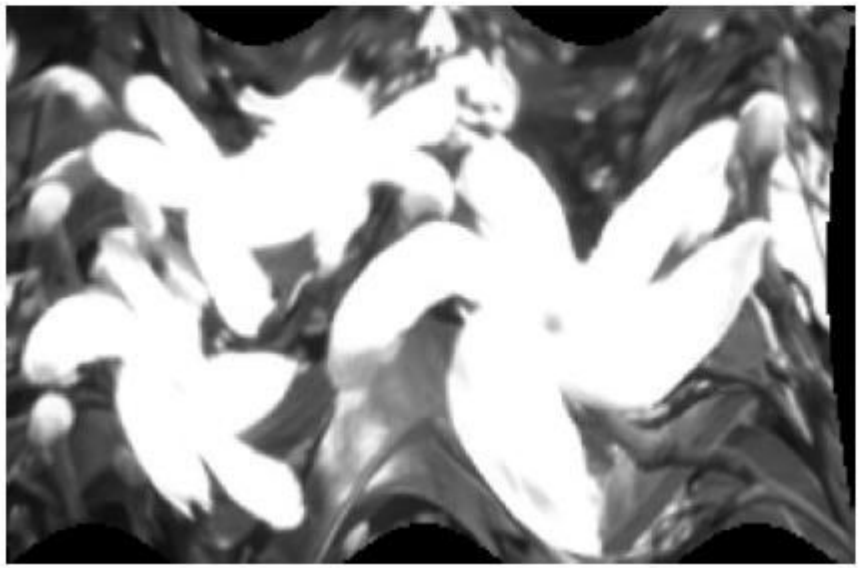

Fig. 6.4 Image with distortion

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Blur

This can mathematically be represented as follows:

$$I_{\text{result}}(x, y) = I_{\text{original}}(x, y) * \sigma(x, y)$$

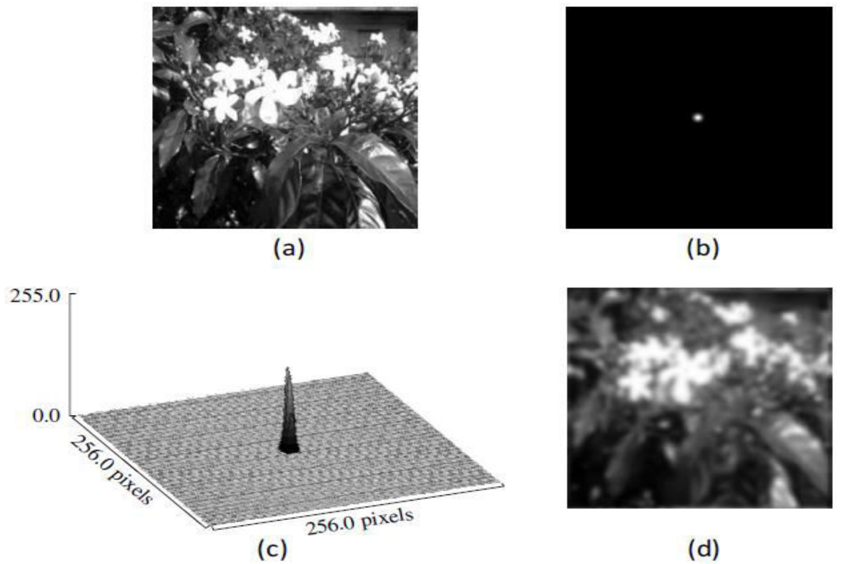

Fig. 6.5 Convolution process (a) Original image (b) PSF (c) Surface plot of PSF (d) Blurred image

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Modulation Transfer Function

$$\text{MTF}(f) = \frac{M_{\text{out}}(f)}{M_{\text{in}}(f)}$$

$$\text{Modulation (contrast)} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

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Image Degradation Model

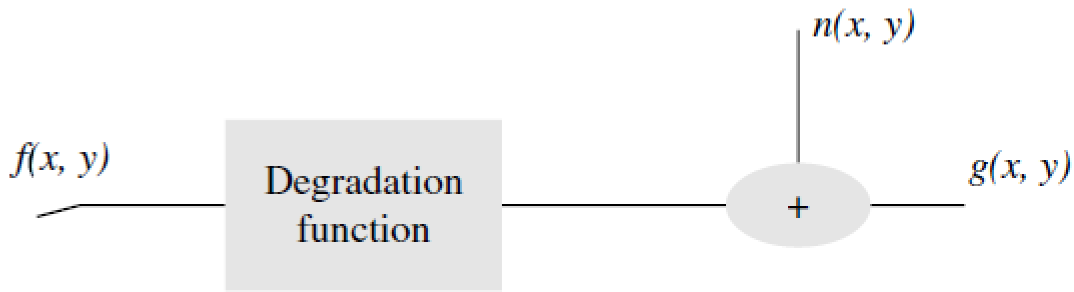

Fig. 6.8 Image degradation model

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Noise

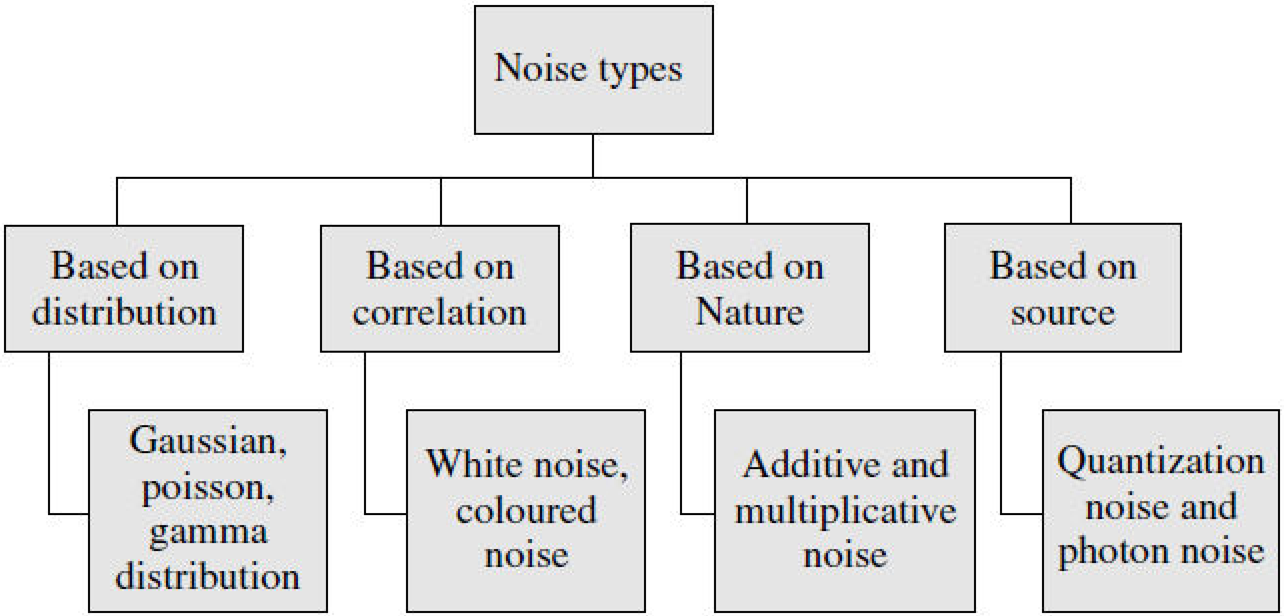

Fig. 6.9 Types of noises

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Gaussian Noise

$$F = f(x, y) \pm N_a$$

where N_a is the Gaussian PDF and $f(x, y)$ is the noiseless image.

(a)

(b) (c) (d)

Fig. 6.10 Gaussian noise (a) Original image (b) Image with Gaussian noise (default variance = 0.01) (c) Image with Gaussian noise (mean = 0.5, variance = 0.01) (d) Image with Gaussian noise (mean = 0, variance = 0.07)

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Salt and Pepper Noise

$$F = f(x, y) \pm N_a$$

where N_a is the Gaussian PDF and $f(x, y)$ is the noiseless image.

(a) (b) (c)

Fig. 6.11 Salt-and-pepper noise (a) Image with default noise density (b) Image with noise density = 0.2 (c) Image with noise density = 0.3

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Exponential Noise

$$P(z) = \begin{cases} a \times e^{-az} & \text{for } z \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

The mean and variance are given as $\frac{1}{a}$ and $\frac{1}{a^2}$, respectively.

(a) (b)

Fig. 6.12 Illustration of exponential noise (a) Original image (b) Image with exponential noise

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Gamma Noise

$$P(z)=\begin{cases}\dfrac{a^b \times z^{b-1}}{(b-1)!}e^{-a^2} & \text{for } z \geq 0\\ 0 & \text{otherwise}\end{cases}$$

Fig. 6.13 Image with gamma noise

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Rayleigh Noise

Fig. 6.13 Image with gamma noise

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Gamma Noise

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-\frac{(z-a)^2}{b}} & \text{for } z \geq a \\ 0 & \text{otherwise} \end{cases}$$

Fig. 6.14 Image with Rayleigh noise

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Uniform Noise

$$P(z)=\begin{cases}\dfrac{1}{b-a} & \text{for } a \le f(x,y) \le b \\ 0 & \text{otherwise}\end{cases}$$

Fig. 6.15 Image with uniform noise

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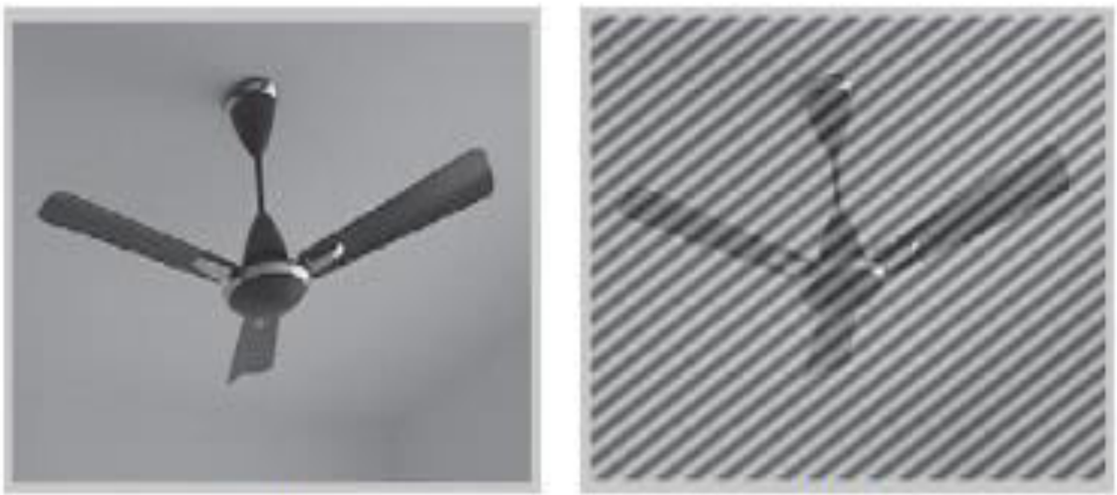
Periodic Noise



(a) (b)

Fig. 5.37 Periodic noise (a) Original image (b) Image with periodic noise

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Blur

$$\begin{pmatrix} & X & \\ & X & \\ & X & \end{pmatrix}, \begin{pmatrix} & & \\ X & X & X \\ & & \end{pmatrix}, \begin{pmatrix} X & & \\ & X & \\ & & X \end{pmatrix}$$

(a)

(b) (c)

Fig. 5.38 Blur and distortion (a) PSF masks (b) Original image (c) Gaussian blur

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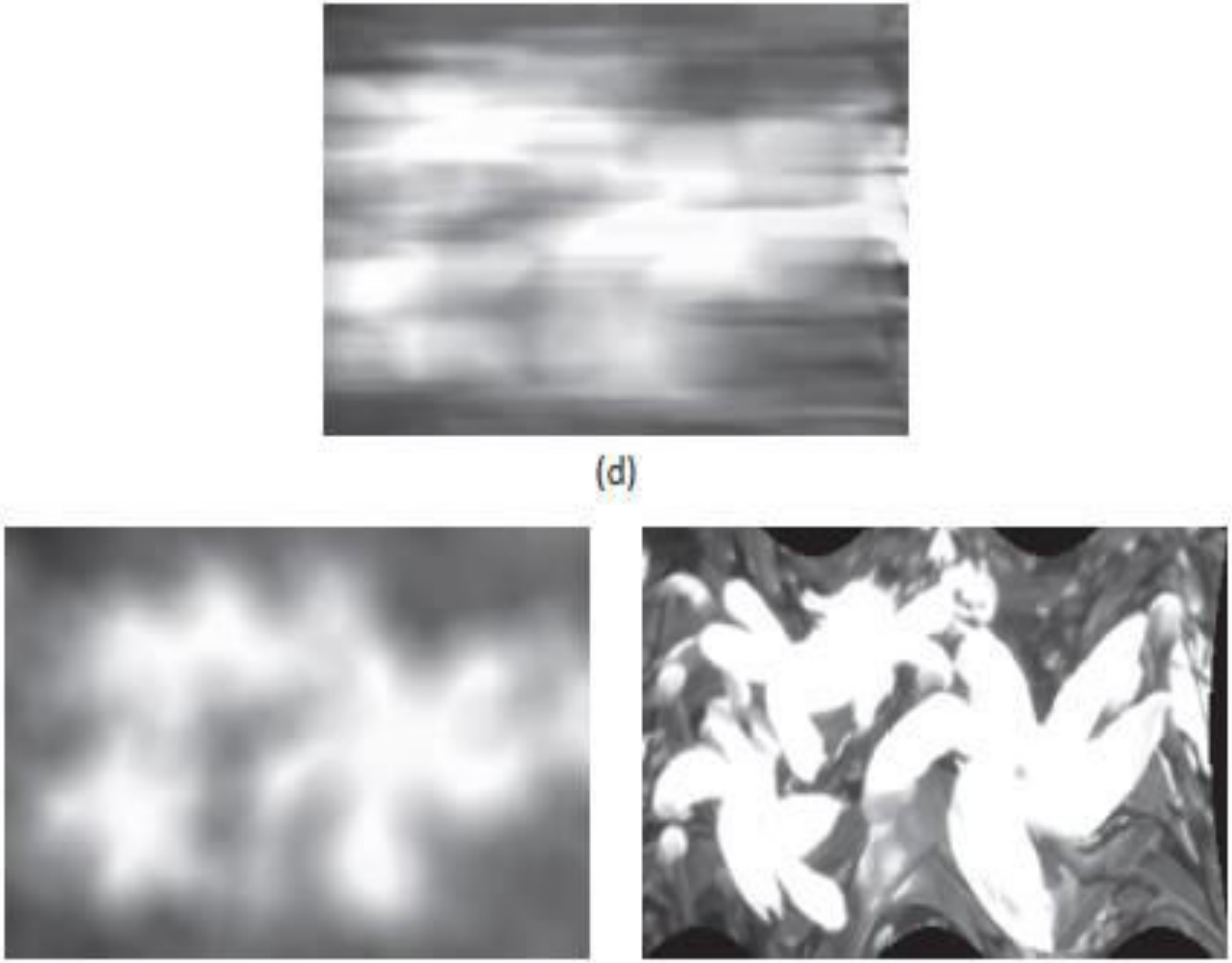

(d) (e) (f)

Fig. 5.38 (Continued) (d) Motion blur (e) Lens out of focus blur (f) Distortion

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Arithmetic Mean Filter

(a) (b) (c)

Fig. 6.17 Application of arithmetic mean filter (a) Original image (b) Image with Gaussian noise (c) Result of arithmetic filter

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Geometric Mean Filter

$$\prod_{(x,y)\in W} f(x,y)^{\frac{1}{N^2}}$$

Fig. 6.19 Application of geometric mean filter (a) Original image with salt-and-pepper noise (b) Result of geometric filter

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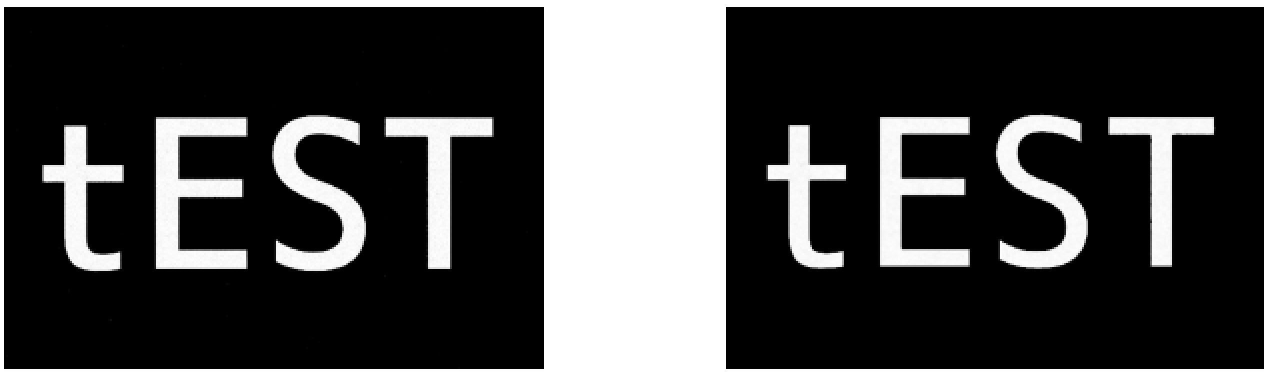
Harmonic Mean Filter

$$\frac{N^2}{\displaystyle\sum_{(x,y)\in W} \frac{1}{f(x,y)}}$$



(a) (b)

Fig. 6.20 Application of harmonic mean filter (a) Original image with Gaussian noise (b) Result of harmonic mean filter

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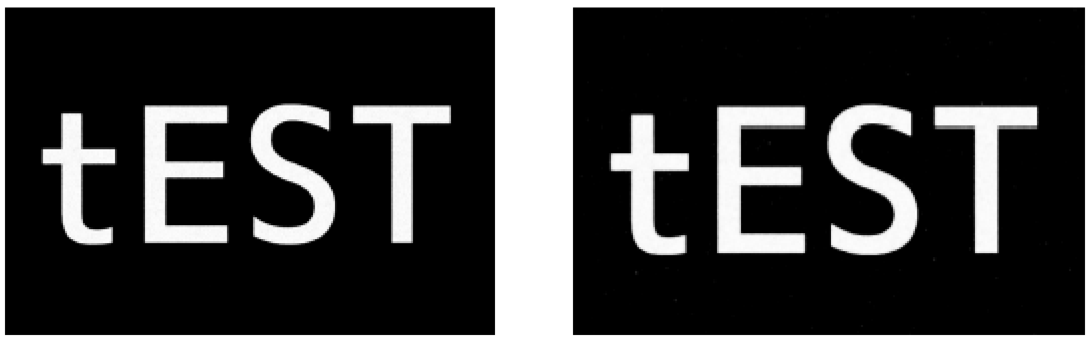
Contra - Harmonic Mean Filter

$$\frac{\sum_{(x,\,y)\in W}^{n} f(x,y)^{R+1}}{\sum_{(x,\,y)\in W}^{n} f(x,y)^{R}}$$



(a) (b)

Fig. 6.21 Application of contra-harmonic mean filter (a) Original image with salt-and-pepper noise (b) Result of contra-harmonic mean filter

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Yp-Mean Filter

Fig. 6.21 Application of contra-harmonic mean filter (a) Original image with salt-and-pepper noise (b) Result of contra-harmonic mean filter

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Contra - Harmonic Mean Filter

$$\frac{\sum_{(x,y)\in W}^{n} f(x,y)^{R+1}}{\sum_{(x,y)\in W}^{n} f(x,y)^{R}}$$

(a) (b)

Fig. 6.22 Application of Yp-mean filter (a) Original image with Gaussian noise (b) Resultant image with Yp-mean filter

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Order Statistic Filter

Let us assume a mask of size 3 × 3. The pixel values are arranged in ascending order $I_1 \leq I_2 \leq \ldots \leq I_9$ based on the grey scale value. The order determines the value that should replace the central pixel.

12	13	15
17	14	18
19	20	21

Using the application of order statistics, this is arranged as

$$\{12, 13, 14, 15, 17, 18, 19, 20, 21\}$$

↓ Minimum ↓ Median ↓ Maximum

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Median Filter

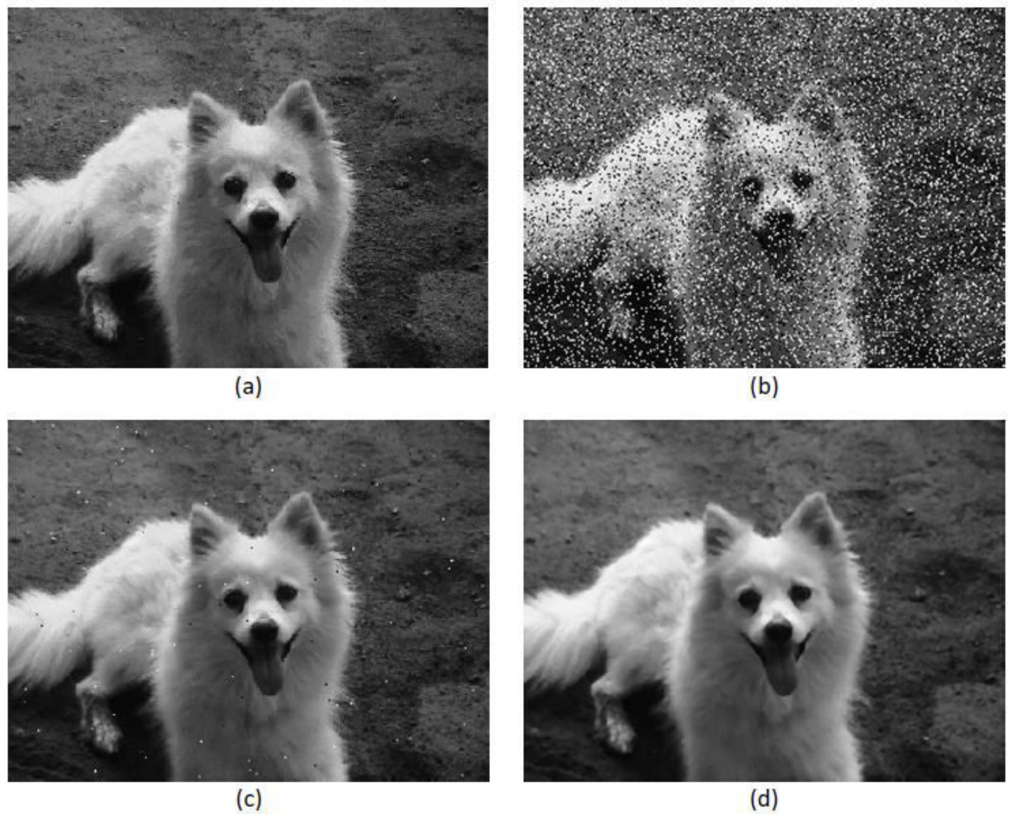

Fig. 6.24 Median filter (a) Original image (b) Image with salt-and-pepper noise (c) Image produced by a median filter with 3 × 3 mask (d) Image produced by a median filter with 5 × 5 mask

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Different Types of Order Statistic Filters

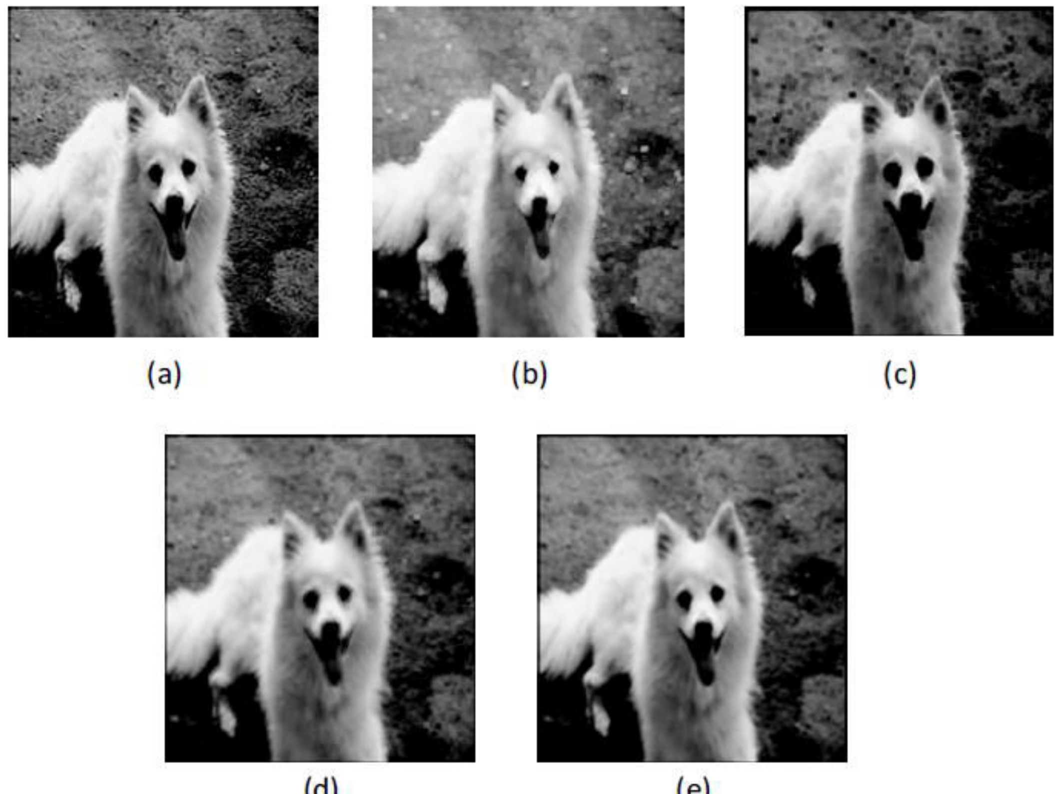

Fig. 6.25 Application of order-statistic filters (a) Original image (b) Output with max filter (c) Output with min filter (d) Output with midpoint filter (e) Output with alpha-trimmed filter

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Periodic Noise


(a)

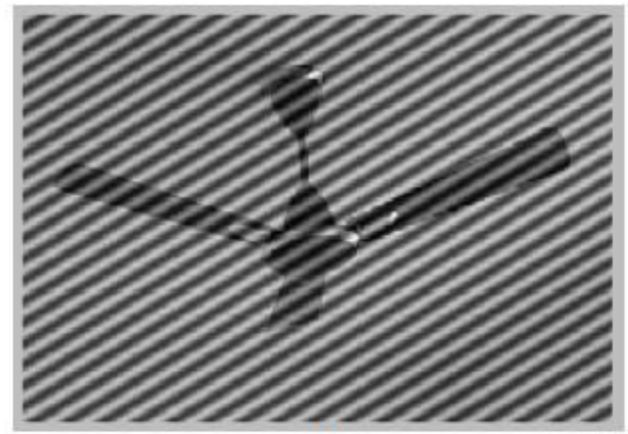
(b)

Fig. 6.26 Illustration of periodic noise (a) Original image (b) Image with periodic noise

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Example

Example 6.4 Consider the image {12, 13, 14, 15, 17, 18, 19, 20, 12}. Find the pixel value to replace the central pixel in case of a median, a max, a min, a midpoint, and an alpha-trimmed filter.

Solution For a median filter, the median value of the sorted list is underlined:

{12, 13, 14, 15, <u>17</u>, 18, 19, 20, 21}

The median filter replaces the central pixel with this median value.

For the given example, the minimum value of the sorted list is underlined:

{<u>12</u>, 13, 14, 15, 17, 18, 19, 20, 21}

The max value in the list is 21. Therefore, the min and max filters replace the central pixel with values 12 and 21, respectively.

For the given example, the mid value is (12 + 21)/2 = 33/2 = 16.5, which is approximately 17. Hence, the midpoint filter replaces the central pixel with 17.

For an alpha-trimmed filter, let us assume that $T = 2$, so two pixel values on either end are excluded in the sorted list to yield the following list:

{14, 15, 17, 18, 19}

Now, the average can be calculated as 83/5. This is approximately 17. Therefore, the earlier central pixel value is not affected.

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Band-pass Filter

The transfer function for a 2D band-pass filter is given as follows:

$$H(u,v)=\begin{cases}1 & \text{if } D_0-\dfrac{W}{2}\le D(u,v)\le D_0+\dfrac{W}{2}\\ 0 & \text{otherwise}\end{cases}$$

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Butterworth Band-pass Filter

$$H_{\text{bp}}^{\text{Butterworth}}(u,v) = \frac{\left[\dfrac{D(u,v)w}{D^2(u,v)-D_0^2}\right]^{2n}}{1+\left[\dfrac{D(u,v)w}{D^2(u,v)-D_0^2}\right]^{2n}}$$

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Gaussian Band-pass Filter

The Gaussian formulation of a band-pass filter is given as follows:

$$H_{\text{bp}}^{\text{Gaussian}}(u,v) = e^{-\frac{1}{2}\left[\frac{D^2(u,v)-D_0^2}{D(u,v)w}\right]^2}$$

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Band-Reject Filters

$$H(u,v)=\begin{cases}0 & \text{if } D_0-\frac{W}{2}\leq D(u,v)\leq D_0+\frac{W}{2}\\ 1 & \text{otherwise}\end{cases}$$

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Butterworth Band-Reject Filter

$$H_{\text{br}}^{\text{Butterworth}}(u,v) = \frac{1}{1 + \left[\dfrac{D(u,v)w}{D^2(u,v) - D_0^2}\right]^{2n}}$$

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Gaussian Band-Reject Filter

$$H_{\mathrm{br}}^{\mathrm{Gaussian}}(u,v) = 1 - e^{-\frac{1}{2}\left[\frac{D^2(u,v) - D_0^2}{D(u,v)w}\right]^2}$$

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Band- Reject Filters

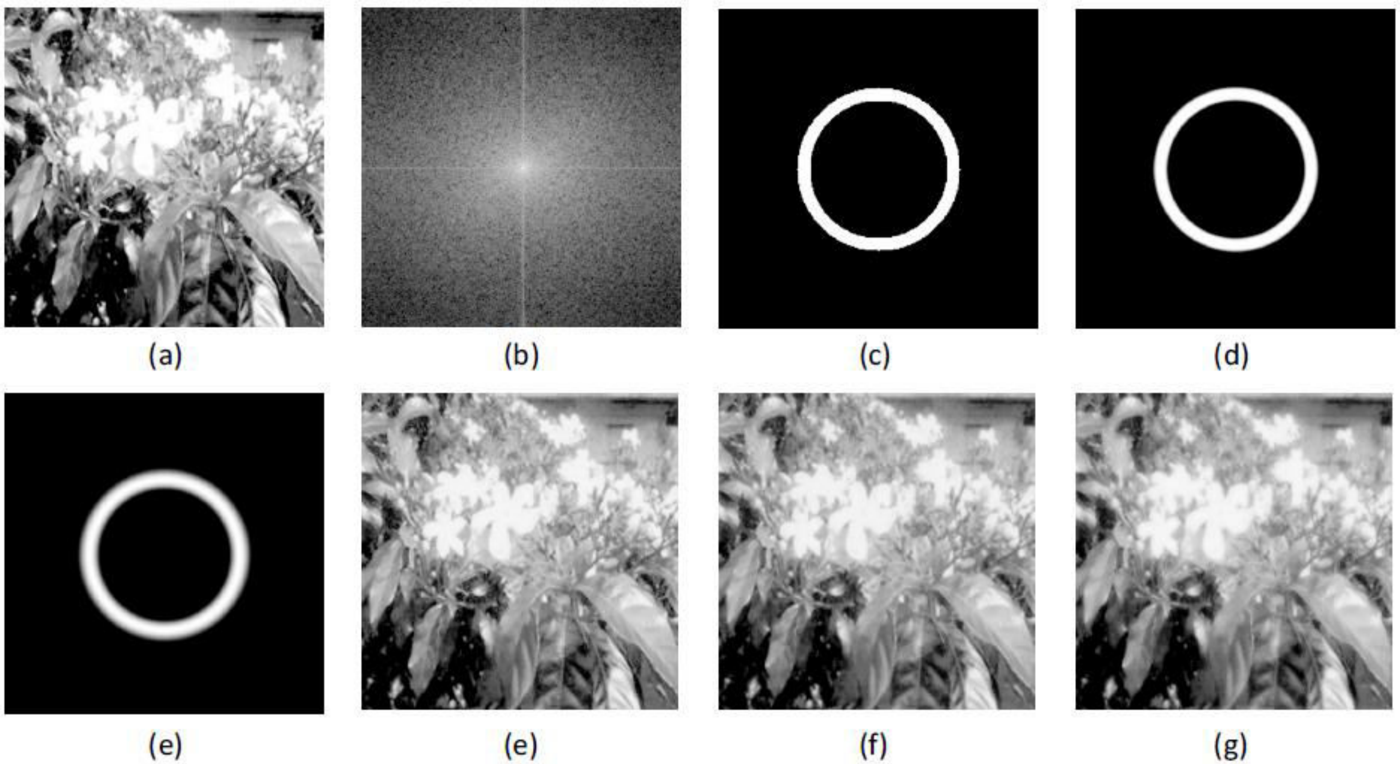

Fig. 6.29 Application of band-reject filter (a) Original image (b) FFT of original image (c) FFT of ideal band-reject filter (d) FFT of Butterworth band-reject filter (e) FFT of Gaussian band-reject filter (f) Resultant image of ideal band-reject filter (g) Resultant image of Butterworth band-reject filter (h) Resultant image of Gaussian band-reject filter

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Notch Filter

$$H_{\text{nr}}^{\text{ideal}} = \begin{cases} 0 & \text{if } D_1(u,v) < D_0 \text{ or } D_2(u,v) > D_0 \\ 1 & \text{otherwise} \end{cases}$$

Here,

$$D_1(u,v) = \left[\left(\left(u - \frac{M}{2} \right) - u_0 \right)^2 + \left(\left(v - \frac{N}{2} \right) - v_0 \right)^2 \right]^{\frac{1}{2}}$$

$$D_2(u,v) = \left[\left(\left(u - \frac{M}{2} \right) - u_0 \right)^2 + \left(\left(v - \frac{N}{2} \right) - v_0 \right)^2 \right]^{\frac{1}{2}}$$

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Butterworth Notch Filter

A Butterworth notch filter of order n is given as follows:

$$H_{\text{nr}}^{\text{Butterworth}}(u,v) = \frac{1}{1 + \left[\dfrac{D_0^2}{D_1(u,v)D_2(u,v)}\right]^n}$$

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Gaussian Notch Filter

A Gaussian notch filter is given as follows:

$$H_{\text{nr}}^{\text{Gaussian}}(u,v) = 1 - e^{-\frac{1}{2}\left(\frac{D_1(u,v)D_2(u,v)}{D_0^2}\right)}$$

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Image Restoration Methods

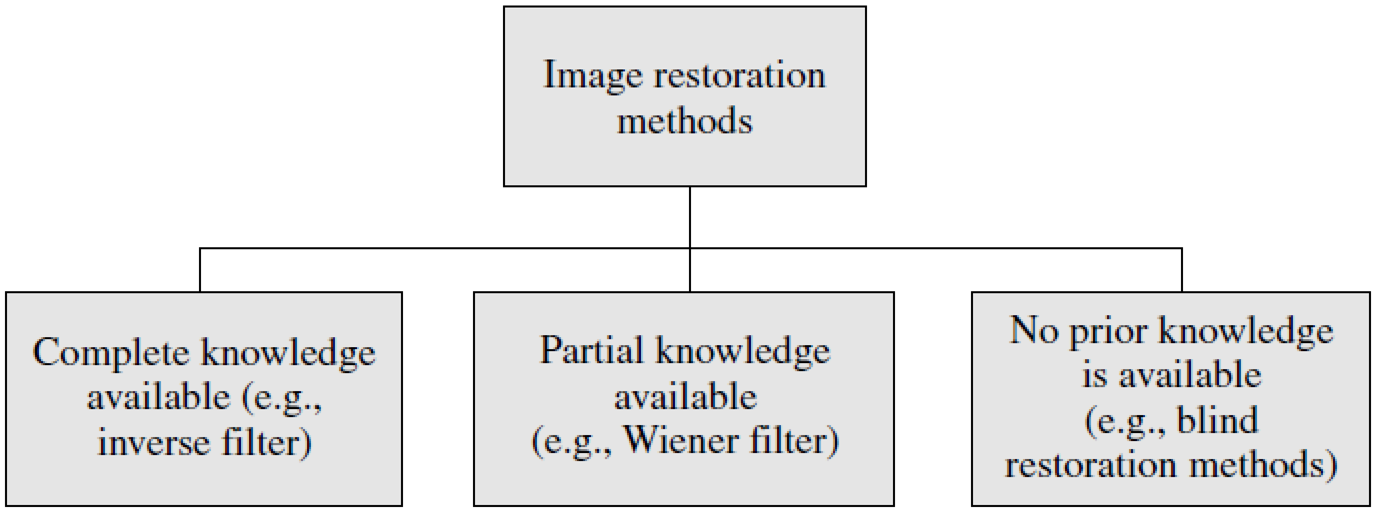

Fig. 6.30 Types of image restoration methods

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Unconstrained Method

$$g = Hf + n$$

Therefore, $n = g - Hf$

$$\begin{aligned}\hat{f} &= \left(H^T H\right)^{-1} H^T g \\ &= H^{-1} g\end{aligned}$$

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Constrained Method

$$J(\hat{f}) = \left(\|Q\hat{f}\|^2 + \alpha\left(\|g - H\hat{f}\| - \|n\|^2\right)\right)$$

$$\hat{f} = \left(H^T H + \gamma Q^T Q\right)^{-1} H^T g$$

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Inverse Filter

$$F(u,v) = \frac{1}{H(u,v)} G(u,v)$$

(a) (b) (c)

Fig. 6.31 Application of inverse filter (a) Original image (b) Image with distortion (c) Restored image using inverse filter

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Wiener Filter

This estimation in frequency domain corresponds to

$$F(u,v)=\left[\frac{1}{H(u,v)}\cdot\frac{\left|H(u,v)\right|^2}{\left|H(u,v)\right|^2+\gamma\left|\frac{S_n(u,v)}{S_f(u,v)}\right|}\right]\cdot G(u,v)$$

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Wiener Filter

$$F(u,v) \cong \left[\frac{1}{H(u,v)} \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + K}\right] \cdot G(u,v)$$

In the absence of any knowledge about noise, K can be assumed as the inverse of the SNR of the image, which is averaged over all frequencies. The value of γ is very crucial and it should be tuned so that $\|n\|^2 = \left\|g - H\hat{f}\right\|^2$.

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Constrained Least Square Filter

$$\widehat{F}(u,v)=\left[\frac{H^*(u,v)}{\left|H(u,v)^2\right|+\gamma\left|p(u,v)\right|^2}\right]G(u,v)$$

1. Specify an initial value of γ.
2. Compute $\widehat{f}$ and $||r||^2$.
3. Check whether $R(u, v) = G(u, v) - H(u, v)\ \widehat{F}(u, v)$; if yes, then stop.
   If $||r||^2 < ||n||^2$, then increase the value of γ.
   Else, if $||r||^2 > ||n||^2 + a$, then decrease the value of γ.

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Iterative Image Restoration

$$f_{k+1} = f_k + (g - h * f_k)$$

$$f_{k+1} = f_k \bullet \left(h \bullet \frac{g}{h \bullet f_k} \right)$$

(a) (b) (c)

Fig. 6.33 Application of iterative filter (a) Original image (b) Degraded image (c) Image restored using Lucy-Richardson filter

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Blind Image Restoration

(a) (b) (c)

Fig. 6.34 Blind image restoration (a) Original image (b) Blurred image (c) Result of blind convolution

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Geometric Transformations

Geometrical transformations include spatial transforms and image interpolation algorithms. If the pixels can be rearranged, spatial transforms are useful. Let us assume that (x, y) are the original coordinates and (x', y') is the deformed image. The spatial transformation can then be represented as

$$x' = T_1(x, y)$$

and

$$y' = T_2(x, y)$$

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The transformations are given by

$$T_1(x, y) = c_1 x + c_2 y + c_3 xy + c_4$$

and

$$T_2(x, y) = c_5 x + c_6 y + c_7 xy + c_8$$

These equations can be solved for the unknown coefficients, to get the transformation function for the given requirement.

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SUMMARY

- Images are degraded by noise, blurs, and artefacts. Image restoration is the process of recovering the original image from a degraded image.
- Degradations can be noise, blurs, and artefacts.
- The impulse response of a system is called the point spread function (PSF). The Fourier transform of the PSF is called the optical transfer function (OTF). The amplitude of an OTF is called the modulation transfer function, which is an indicator of the performance of an imaging system.
- Distortions and artefacts are extreme intensity and colour imperfections that make images meaningless.
- Distortions can be modelled as image degradation models. Degradations can be estimated by observation, experimentation, and modelling.
- Noise is a disturbance. Noises can be classified based on their distribution, correlation, nature, and source.
- Mean filters and order-statistic filters are used to restore an image in the presence of noise only.
- Mean filters compute the average of the pixels that fall within the selected window and replace the central pixel. The different types of mean filters are arithmetic mean filters, geometric mean filters, harmonic filters, contra-harmonic filters, and Yp-mean filters.
- An order-statistic filter is a non-convolution-based filter that orders the pixels within the selected window and selects a value that replaces the central pixel.
- Order-statistic filters include median, maximum, minimum, midpoint, and alpha-trimmed filters.
- Band-pass filtering is a combination of both low- and high-pass filters. A band-pass filter allows all frequencies except those present in the narrow band defined by the cut-off values. On the other hand, a band-reject filter attenuates the frequencies that falls within the band.
- An inverse filter removes blurs and noise using the deconvolution process.
- The two types of algebraic methods used for image restoration are unconstrained and constrained methods.
- A Wiener filter is an optimal filter. It finds an estimate $\hat{f}(x, y)$ of the original image $f(x, y)$ using the least mean square.
- The iterative approach recovers the original image from a degraded image in successive iterations.
- In situations where there is little or no prior knowledge of blurs, blind image restoration methods are useful. Here, restoration is carried out by estimating the blurring functions either directly or indirectly.
- Geometrical transforms include spatial and image interpolation algorithms.

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