

Digital Image Processing

S. Sridhar, Associate Professor,
Department of Information Science and
Technology, College of Engineering
Guindy Campus, Anna University,
Chennai



Chapter 10 Image Features Representation and Description

What is a feature?

- Any characteristic or primitive of an object that helps to distinguish or discriminate an object from other objects is called an image feature.
- Natural features These are visual appearances of the image that are natural to the object, such as brightness and texture.
- Artificial features These are derived features that are obtained using image manipulations. Amplitude histograms and frequency spectrums are examples of this category

Feature Extraction

 Feature extraction is a process of extraction and generation of features to assist the task of object classification. This phase is critical because the quality of the features influences the classification task.

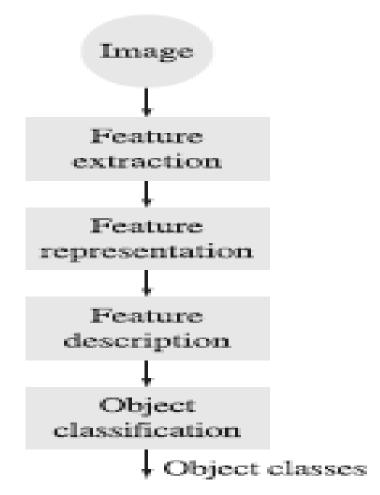


Fig. 10.1 Interaction of feature extraction with the object classification process

Characteristics of a Good Feature

Shift invariance This is the ability of the feature to remain constant when shift operations are performed.

Rotation invariance This is the ability of the feature to remain constant when rotated.

Size invariance This is the ability of the feature to remain constant when its size is changed.

Mirror, shear, and affine invariance Features that remain constant even if operations such as mirroring, shear, and affine transforms are applied, are mirror invariant, shear invariant, and affine invariant, respectively.

- **Occlusion invariance** When all or some parts of the object are hidden, the property of the features that do not change is said to be occlusion invariant.
- **Discrimination** The properties should distinguish one object from the other and there should be no overlapping features.
- Reliability The values should be reliable, that is, similar objects should have similar values.
- Independence The features are said to be independent if they are statistically uncorrelated from each other. In other words, change in one feature value does not affect the values of the other features.
- Resistance to noise Agood feature should be immune (resistant) to noise, artefacts, etc.

 Compactness The features should be in small numbers so that they can be represented compactly.

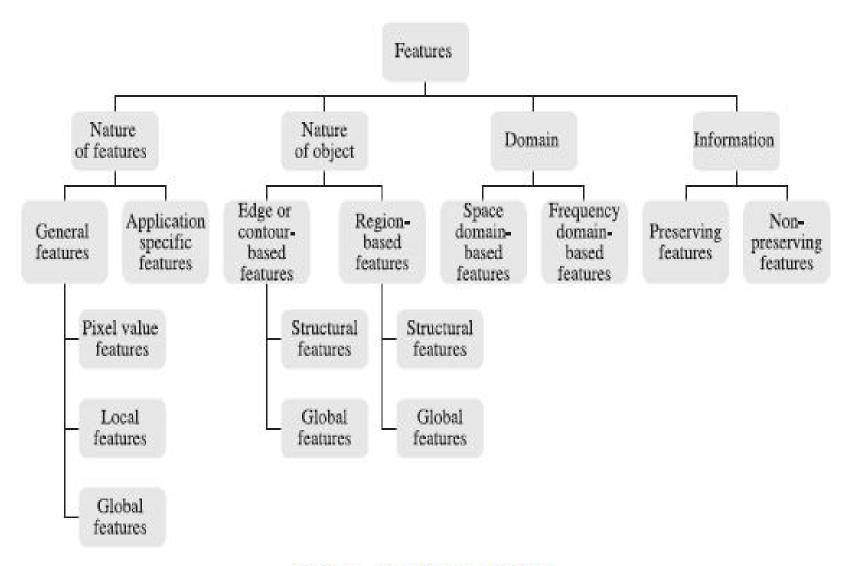


Fig. 10.2 Classification of features

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BOUNDARY REPRESENTATION

Chain Code

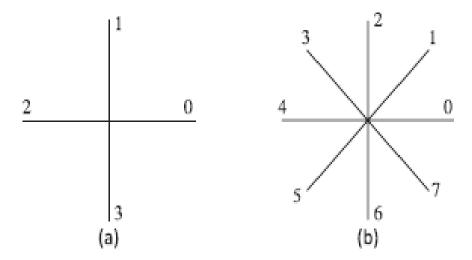


Fig. 10.3 Image chain codes (a) 4-Directional code (b) 8-Directional code

Tracking Process

The algorithm for the tracking process of 4-neighbourhood is given as follows:

- 1. Start scanning the image from left to right looking for object pixels. Start with the object pixels that are marked 1. Call it P_0 . Set d = 3. d is called the direction variable.
- 2. Label the pixel as the current pixel. The previously visited pixel is labelled as previous.
- 3. The next pixel is determined by searching the neighborhood in an anticlockwise manner. Search the neighborhood in the anticlockwise direction using $(d + 3) \mod 4$.
- 4. Label the current pixel as previous pixel and label the next pixel as current. Update d.
- If the current boundary pixel P₀ is equal to P_{n-1}, that is, if current = previous, then stop; otherwise, go back and repeat the process till the start pixel is reached.
- 6. Label the region. Display $P_0 \dots P_{n-1}$ and exit.

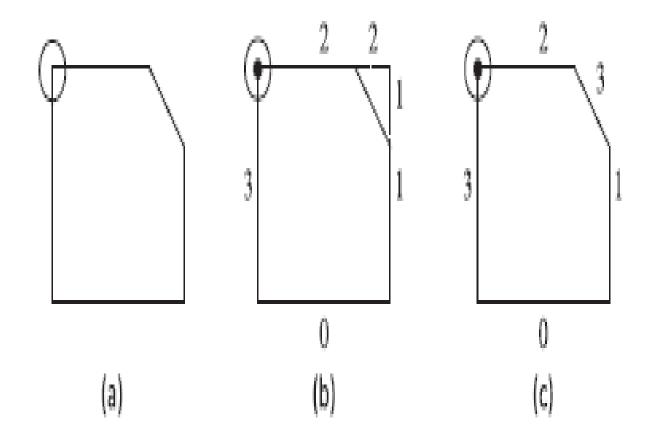


Fig. 10.4 Chain code (a) Original image (b) 4-Directional chain code 301122 (c) 8-Directional chain code 30132

Advantages of Chain code

Some of the advantages of the chain code are as follows:

- Chain codes preserve the information of interest.
- A chain code is a compact representation of an image.
- A chain code is a prerequisite for any image analysis operation.
- Chain codes are translation-invariant, but the same thing can not be always said for rotational and scale invariance.
- Chain codes also indicate the corner points. If there is a consecutive change between chain codes, it indicates a change of direction and this represents a corner point.

Disadvantage

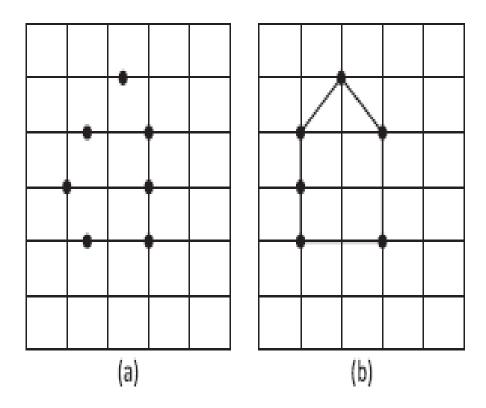


Fig. 10.5 Resampling process (a) Original image points (b) Resampled image

DCC

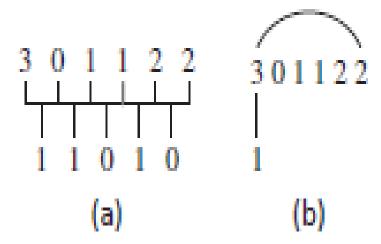


Fig. 10.6 Normalization process

(a) First difference (b) Circular chain code

Polygonal Approximation

- Divide the boundary into small segments and fit a line for each segment.
- Draw a line from any point to its farthest point. Identify the longest line segment that connects two farthest points. It can be observed in Fig. 10.7, two points Pi and Pj are chosen and connected by a line segment.
- Compute the perpendicular distance from the boundary to the line segment.
- Check the perpendicular distance with a threshold. If the threshold is crossed, then the line is split.
- Identify the vertices that are very close to the prominent inflection points of the curve.An inflection point is a point on a curve where the curvature changes its sign.
- Connect the points to get the approximate polygons.
- Repeat the process for the new line until there is no longer a need for a split.

Merging Method

- Choose a contour starting point.
- Add pixels, one-by-one to the line segment.
- Let the line pass through the new pixel.
- Compute the squared error for all the points along the segment.
- If the error is greater than the threshold, begin a new line at the current point.

Signatures

 A signature is a 1D functional representation. It is a plot of the distance from the centroid to the boundary as a function of the angle.

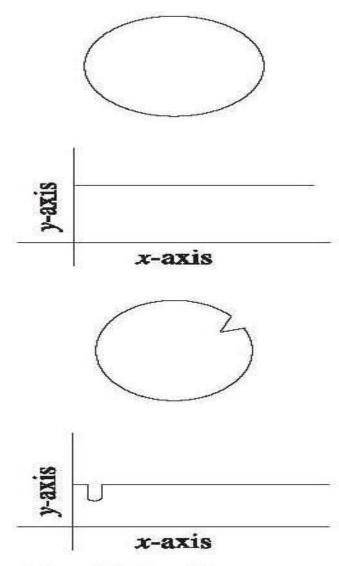


Fig. 10.9 Signature showing the centroidal profile

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Bending Energy

 Boundary curvature is the differential of the y-s curve, i.e., the curvature at every point in the boundary. The integration of all squared curvature values along the entire contour gives a single descriptor called bounding energy (or bending energy). This is computed in the discrete domain as the sum of squares of the border curvature c(x) over the boundary of length L.

Bending energy =
$$\frac{1}{L} \sum_{i=1}^{L} c_i^2(x)$$

Statistical Moments

$$\mu_n(v) = \sum_{i=0}^{A-1} (v_i - m)^n p(v_i)$$

where

$$m = \sum_{i=0}^{A-1} v_i p(v_i)$$

Here m is the mean, μ_n is the variance, and A is the number of amplitude increments used for splitting the amplitude scale.

Region Representation

- 1. Region decomposition
- 2. Bounding regions
- 3. Internal features

Thinning Algorithms

The eight neighbours of the border point are shown in Fig. 10.10.

```
x9 x2 x3
x8 x1 x4
x7 x6 x5
```

Fig. 10.10 Sample image for obtaining the skeleton

Phase 1

- Find the non-zero neighbours of the point x1. Check if they are between 2 and 6.
- Find the number of zero to one transitions in the sequence x2 x3... x9 x2. Check if its
 value is 1.
- 3. Find if $x2 \cdot x4 \cdot x6 = 0$
- 4. Find if $x4 \cdot x6 \cdot x8 = 0$

Contd...

Phase 2

- 5. Find the non-zero neighbours of the point x1 again. Check if they are between 2 and 6.
- 6. Find the zero to one transition in the sequence x2 x3... x9 x2. Check if its value is 1.
- 7. Find if $x2 \cdot x4 x8 = 0$
- 8. Find if $x2 \cdot x6 \cdot x8 = 0$

BOUNDARY DESCRIPTIONS

Boundary length The number of pixels present in the boundary roughly gives the boundary length. Object length can be obtained using chain codes and is given as

Length = Number of vertical components + Number of horizontal components + $\sqrt{2}$ × Number of diagonal components

Diameter Let $D(p_i, p_j)$ be the line connecting two points p_i and p_j on the boundary. The maximum distance connecting any two points p_i and p_j is called the diameter.

The diameter is given as

$$Diameter = \max_{i,j} |D(p_i, p_j)|$$

Curvature The rate of change of slope is called curvature. If the change of slope is non negative, the object is called convex, otherwise it is called concave. Further, it can be characterized as corner if the curvature exceeds 90°.

Shape Number

- 1. Find the major axis and minor axis of the boundary.
- 2. Using the major and minor axes, the sampling boundary is constructed.
- 3. Resampling process is carried out and the chain code is obtained.
- 4. Normalization is carried out by obtaining the first difference.
- 5. The shape number is obtained by obtaining the minimum magnitude of the first difference.

Fourier Descriptors

The procedure of obtaining Fourier descriptors is illustrated as follows:

- Take N digital points of the boundary, where N represents the length of the closed curve.
- Apply the Fourier transform to the boundary points.

$$s(u) = \frac{1}{N} \sum_{k=0}^{N-1} s(k) e^{-\frac{j2\pi ux}{N}}$$
, for $u = 0, 1, 2, ..., N-1$

The complex coefficients of s(u) are called Fourier descriptors.

The inverse transform can be applied to obtain the original points.

$$s(k) = \frac{1}{N} \sum_{u=0}^{N-1} s(u) e^{\frac{f2\pi ux}{N}}$$
, for $k = 0, 1, 2, ..., N-1$

4. However, all the points are not required. Some of the boundary points can be dropped. This leads to loss of some information. Thus the inverse transform leads to an approximation of the boundary given as

$$\hat{s}(k) = \frac{1}{N} \sum_{n=0}^{N-1} s(u)e^{\frac{j2\pi ux}{N}}$$
, for $k = 0, 1, 2, ..., N-1$

Run-length Code

Run = <row, column, length>

Projections

Projection is a metric that denotes a binary image as a function of row coordinates. The horizontal projection is defined as

$$P(x) = \sum_{x} f(x, y)$$
 for all x

In a binary image, this is the number of foreground pixels in the horizontal direction.

This is also known as horizontal picture. The vertical projection is given as

$$\sum_{y} f(x,y)$$

The projection along the direction d with respect to x axis can be given as

$$p_{f,\theta} = \sum_{x} f(x, \tan \theta + d)$$

Projections save a large amount of time and space. A sample image and its projections are shown in Fig. 10.12.

Fig. 10.12 Binary image and its projections

Concavity Tree

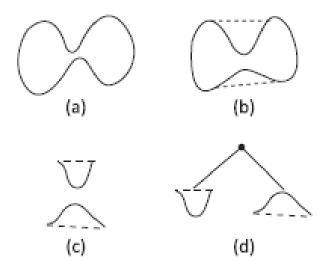


Fig. 10.13 Concavity tree (a) Original polygon (b) Convex hull (c) Deficiency (d) Concavity tree

COMPONENT LABELLING

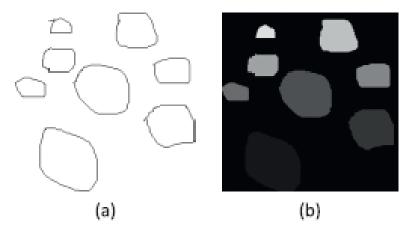


Fig. 10.14 Component labelling (a) Original image (b) Labelled image

Recursive Algorithm

- 1. Initialize the label L to -1.
- 2. Scan the image left to right and top to bottom.
- 3. Initialize a queue Q.
- 4. If the pixel value f(x, y) is greater than zero then insert (x, y) into the queue.
- 5. If the queue is not empty, proceed to step 6; else proceed to step 12.
- 6. Remove the value at the front of the queue and assign it temporarily to (s,t).
- 7. For all the neighbours of (s,t) do steps 8–11.
- 8. Recursively check the neighbour and assign the same label if they are unlabelled.
- 9. Insert it into the queue.
- 10. Increment the label as L = L 1. The labels are negative numbers for distinguishing them from the pixel values.
- 11. Stop when all the pixels with the value '1' are labelled.
- 12. Exit.

Sequential Algorithm

A
B Target(X)

Fig. 10.15 Image with an unlabelled pixel—Target (X)

Sequential Algorithm

Table 10.1 Rules for assigning a label for *X*

Rule	A	В	Target(X) for which a label is to be assigned	Output label
1	0	0	1	Assign a new label L
2	L	0	1	Assign the label L of A to X . Now $X = L$
3	0	L	1	Assign the label of B to X. Now $X = L$
4	M	L	1	This is the case of confusion known as the multiple-label problem and can be resolved in phase 2.

REGIONAL DESCRIPTORS

Histogram (or Brightness) Features

Mean The mean of the histogram is given by

$$Mean(\overline{b}) = \sum_{b=0}^{L-1} b.p(b)$$

The mean indicates the brightness of the image.

Standard deviation The standard deviation is given by

$$SD = \sigma_b = \sqrt{\sum_{b=0}^{L-1} [(b - \overline{b})^2 p(b)]^{\frac{1}{2}}}$$

Standard deviation indicates the contrast. This specifies the spread of data.

Skewness is given by

Skewness =
$$\frac{1}{\sigma_b^3} \sum_{b=0}^{L-1} (b - \overline{b})^3 p(b)$$

This indicates the asymmetry about the mean in the grey level distribution. Skewness can also be calculated as $\frac{\overline{b} - \text{mode}}{\sigma}$.

Kurtosis The kurtosis is given by

Kurtosis =
$$\frac{1}{\sigma_b^4} \sum_{b=0}^{L-1} (b - \overline{b})^4 p(b) - 3$$

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Energy The energy of the image is given as the sum of brightness values of all the pixels present in the object. This is called the zero order spatial moment.

Energy =
$$\sum_{b=0}^{L-1} [p(b)]^2$$

The energy is 1 for an image having a constant intensity and its value becomes smaller when the pixel values are distributed across different grey levels.

Entropy The entropy of the image is given by

Entropy =
$$-\sum_{b=0}^{L-1} p(b) \cdot \log_2 p(b)$$

Additional Features

Colour features Similar to brightness, colour features are very useful in object characterization. Colour histograms can be obtained for colour images and colour moments, which have been discussed in detail in Chapter 8, are very useful features of colour images.

Region density (or densitometric) features The extraction of density measures requires the original and the segmented images and is represented in terms of grey scale values.

Some of the useful density measures are

Transmission (T) This is defined as a measure of proportion of incident light that passes through the object.

Optical density (00) This is defined as a measure that indicates absorption. This measure is the opposite of the transmission measure and is given as

$$OD = \log_{10}\left(\frac{1}{T}\right) = -\log_{10}T$$

Integrated optical density (100) This is a measure given as the weighted sum of all grey level bins of the histogram. This is given as $\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)$. In terms of histogram, it

Contd...

can be given as $\sum_{x=0}^{L-1} r \times H(r)$. Here M and N refer to the size of the image and H(r) is the histogram of the image for grey level r. L is the total number of grey levels of the image.

Shape Features

 Shape is one of the most important features of an object.

- Shapes can be generically classified for further analysis into two types as
 - 1.2D shapes 2. 3Dshapes

Geometrical Features

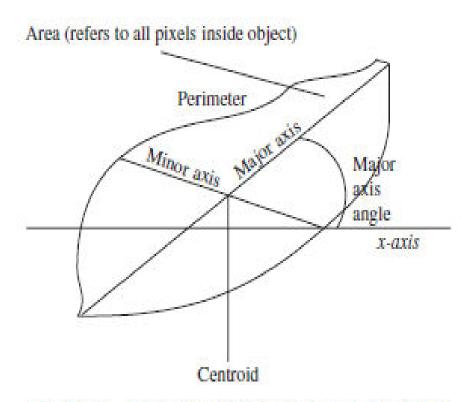


Fig. 10.17 Geometrical features of a sample object

Area

- Set region area = 0.
- Read the next character c.
- 3. If c is not the end of file, do steps 4-8; otherwise go to step 9.
- 4. If c = 0, then area = area vertical position.
- 5. If c = 1, then area = vertical position + 1.
- 6. If c = 3, then area = vertical position -1.
- 7. If c = 2, then area = area + vertical position.
- 8. Go to step 2.
- The area of the object is the region area.
- Exit.

Area

Table 10.2 Contributions of 8-directional chain code towards area calculation

Chain code	Contribution towards area
0	Area + y
1	Area + $(y + 0.5)$
2	Area
3	Area $-(y - 0.5)$
4	Area – y
5	Area $-(y - 0.5)$
6	Area
7	Area + $(y - 0.5)$

Perimeter

For a 4-directional code: P = Number of chain codes

and

for an 8-directional code: $P = \text{Even count} + \sqrt{2} \text{(odd count)}$ units

Shape factor This is also known as compactness and is given as

Circularity =
$$\frac{(Perimeter)^2}{Area}$$
 or $\frac{P^2}{A}$

Area to perimeter ratio Another useful measure is the area to perimeter ratio which is defined as $\frac{A}{P}$. This metric also indicates the roundness, circularity, or thinness ratio. This metric indicates how close an object is to a circle.

$$\frac{A}{P} = 4\pi \times \text{Area/Perimeter}^2$$

This is a dimensionless quantity and ranges between 0 and 1. If its value is 1, then the object is a perfect circle. This measure is insensitive to scaling transformations.

Object length (major axis) The longest line that can be drawn through the object connecting the two farthest points in the boundary is called its major axis. If the major axis end points are (x_1,y_1) and (x_2,y_2) , then the length of the object is the major axis length, and can be calculated as

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

The orientation of the object can be obtained using the angle between major axis and x-axis of the image and is described as

$$\tan^{-1}\left(\frac{(y_2-y_1)}{(x_2-x_1)}\right)$$

The major axis and major axis angle are shown in Fig. 10.17.

Object width Object width can be calculated using the minor axis. Minor axis is the longest line that can be drawn through the object while maintaining perpendicularity with the major axis. This is calculated using the formula

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

where (x_1, y_1) and (x_2, y_2) are the end points of the minor axis.

Boundary Box Area

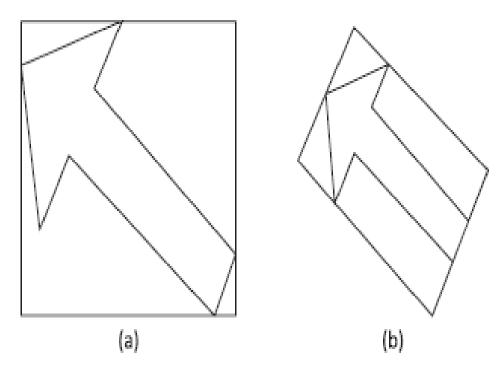


Fig. 10.18 Boundary box area (a) Feret box (b) Minimum bounding rectangle

Parameters

Boundary box area = Major axis length \times Minor axis length

The following features can be derived from the minimum bounding rectangle.

Elongatedness is described as

$$\frac{\text{Elongatedness} = \frac{\text{Length of the major axis}}{\text{Perimeter}}$$

It is also known as aspect ratio and is expressed as

Length of the region bounding rectangle

Width of the region bounding rectangle

The rectangularity is given as

$$F = \frac{\text{Region area}}{\text{Area of the minimum bounding recangle}}$$

Spatial moments

Zero order spatial moment The circle has a centre, whereas in complex objects such as irregular polygons, the centre of the object is indicated by the centroid which is equivalent of a circle's centre. The centre of gravity (centroid or centre of mass) can be specified as

$$x_{c} = \frac{1}{\sum_{x=0}^{N} \sum_{y=0}^{M} f(x,y)} \times \sum_{x=0}^{N} \sum_{y=0}^{M} x f(x,y)$$

$$y_{c} = \frac{1}{\sum_{x=0}^{N} \sum_{y=0}^{M} f(x,y)} \times \sum_{x=0}^{N} \sum_{y=0}^{M} y f(x,y)$$

Spatial moments

Centre of mass The center of mass can be calculated using the following formula:

Centre of mass
$$x = \frac{\text{Sum of the object's } x\text{-pixel coordinates}}{\text{The number of pixels in the object}}$$

Similarly,

Centre of mass
$$y = \frac{\text{Sum of the object's } y\text{-pixel coordinates}}{\text{The number of pixels in the object}}$$

Contd...

Grey centroid This is the balance point of the image with equal brightness in all the directions of the image. Grey centroid (x, y) can be calculated as

$$x = \frac{\text{Sum of (pixel } x \text{ coordinates} \times \text{pixel brightness)}}{\text{Sum of pixel brightness of the object}}$$

$$y = \frac{\text{Sum of (pixel } y \text{ coordinates} \times \text{pixel brightness)}}{\text{Sum of pixel brightness of the object}}$$

It is possible to calculate higher order spatial moments, but generally only the first and second order spatial moments are used.

Central Moments

Central moments are calculated as

$$\mu_y = \sum_{x} \sum_{y} (x - x')^t (y - y')^j f(x, y)$$

The normalized central moments are defined as

$$\eta_{ij} = \frac{\mu_{ij}}{(\mu_{00})^{\lambda}}$$

where

$$\lambda = \frac{(i+j)}{2} + 1$$
 and $(i+j) \ge 2$

For example, the central moment μ_{30} can be calculated as

$$\sum_{x} \sum_{y} (x - x')^{3} (y - y')^{0} f(x, y)$$

Topological Features

Hole A hole is shown in Fig. 10.20.

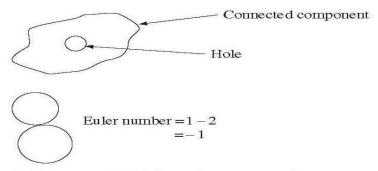


Fig. 10.20 Topological features (a) Hole and a connected component (b) Euler number

It can be observed that the transformations such as stretching, shrinking, and rotation do not affect the presence of the hole.

Connected component The connected component is as shown in Fig. 10.20. It defines the regions that are populated by pixels that share common characteristics.

Euler number This is also known as genus. This is one of the most important topological properties. It is given as

$$E = C - H$$

Here C is the connected component and H is the number of holes. For a polygonal network, it is given as

Euler number =
$$V - E + F$$

Here V is the number of vertices, E is the number of edges, and F is the number of faces. Euler number is shown in Fig. 10.20.

Number of holes present This is a count of the number of holes that are present in the object.

Total hole area This is the total pixel area of the interior holes present in the object.

Total hole area/object area This parameter is a measure of object proliferation. The value of this parameter can range from zero to one. If the value is one, then the entire object itself is a hole.

Number of connected components The number of connected components that are present in the object is also an important topological feature.

Transform Features

- 1. Box
- Horizontal slit
- Vertical slit
- Ring
- Sector

These masks are shown in Figs 10.21(a)-10.21(e). This concept can be extended to include other transforms also

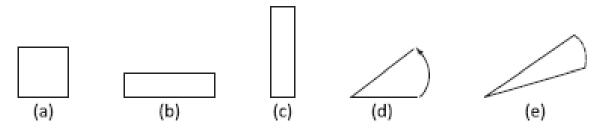


Fig. 10.21 Transform feature masks (a) Box (b) Horizontal slit (c) Vertical slit (d) Ring (e) Sector

Transform Features

$$Power = |F(u, v)|^2$$

Normally, a portion of the spectral element is taken and measured as the spectral region power. The spectral region power is given as

Spectral region power =
$$\sum_{v=0}^{N-1} \sum_{v=0}^{N-1} |F(u,v)|^2$$

Texture Features

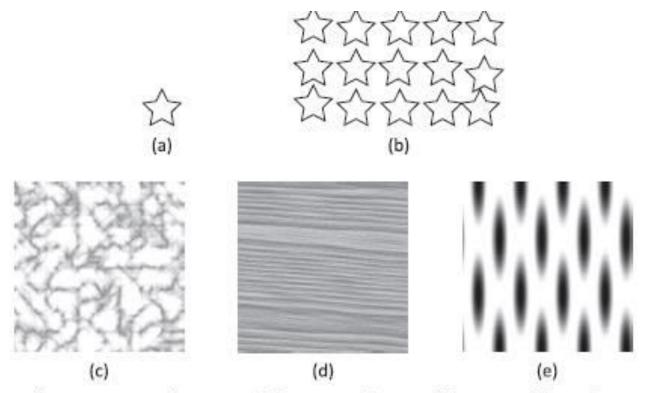


Fig. 10.22 Sample textures (a) Primitive element (b) Texture (c) Random (d) Wood (e) Plasma

Texture analysis

Texture analysis can be done in the following three ways:

Statistical methods Statistical methods can be used to model textures. Grey level co-occurrence matrix is one of determining parameters for texture analysis.

Spectral methods Spectral methods use transforms such as FFT to characterize textures.

Structural methods Spatial relationships among primitives are used to model textures. This method uses formal language theory to analyse and characterize textures.

GLCM

The GLCM of an $N \times M$ image is a 2D matrix. The elements of the matrix are the joint occurrence of intensity levels X and Y at a certain distance d and an angle θ . Hence many occurrence matrices are possible. For example, a sample template, image, and the corresponding GLCM are shown in Figs 10.23(a)–10.23(c).

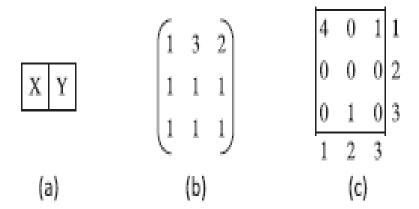


Fig. 10.23 GLCM technique (a) Template (b) General image (c) GLCM

Angular second moment The angular second moment is given as

$$F_1 = \sum \sum C(x, y)$$

where c(x, y) is the co-occurrence matrix.

Contrast This is an indicator of the local variations. For images having uniform intensity, this value is zero. Contrast is given as

$$F_2 = xy(x - y)^2 \sum C(x, y)$$

Correlation This is a measure of the linear dependency of the grey levels of the neighbouring pixels. Therefore it measures the grey tone linear dependence.

Entropy This is a measure of complexity. Complex textures have more entropy than simpler structures. It is measured as

$$F_4 = \sum xy \log_2 C(x, y)$$

Inverse difference moment This indicates the amount of local uniformity. This is calculated as

$$F_5 = \sum \frac{C(x,y)}{(1+(x-y)^2)}$$

Syntactic and Structural Features

- Identification of primitives: Primitive is a fundamental block of an object. It characterizes the fundamental lower level sub-pattern. The primitive can be a square or any other geometrical shape that describes the surface. For example, in a room's floor, if it is tiled, then every tile represents the primitive.
- 2. Grammar construction: The grammar of the pattern can be formally defined as

$$G = \langle V_N, V_T, P, S \rangle$$

Here, V_T is the set of primitives or terminals, V_N is the set of non-terminals, P is the production rule, and S is the starting symbol.

- 3. Let the production rules be of the form $\alpha \to \beta$. Here α and β are the strings.
- The object can be represented as S → aS.

FEATURE SELECTION TECHNIQUES

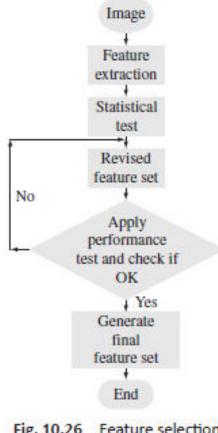


Fig. 10.26 Feature selection process

Principal component analysis (PCA)

To illustrate PCA, a sample dataset is created with the attributes length, breadth, area, perimeter, circularity, and compactness. The data is chosen randomly. Principal component analysis is applied to the dataset and its dimensionality is reduced. It is effective in removing the attributes (or components as per PCA) which do not contribute much. In addition, if the original data is desired, it can be obtained, and therefore no information is lost. Scree plot is a technique to visualize the principal components.

The scree plot of a given sample dataset is shown in Fig. 10.27.

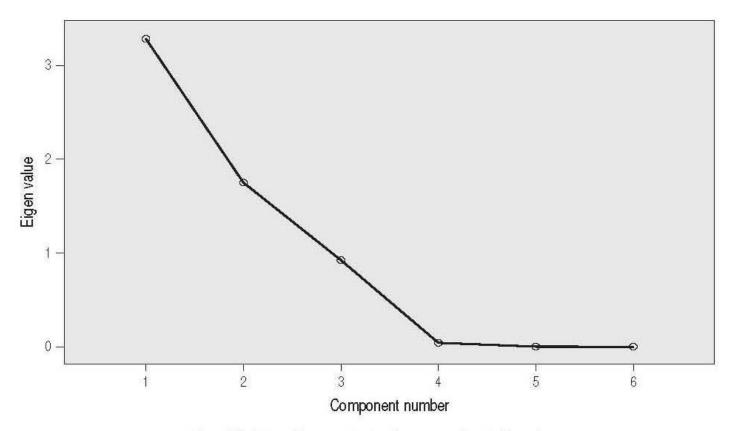


Fig. 10.27 Scree plot of a sample dataset

Summary

- 1. Any characteristic or primitive of an object that helps to distinguish or discriminate it from other objects is called an image feature.
- 2. Feature extraction is a process of extraction and generation of features to assist the object classification task.
- 3. Features can be categorized as pixel, local, or global features. Examples of pixel features are pixel colour and location. Local features are based on a small portion of the image or a neighbourhood. Global features are those that correspond to the entire image, such as statistical and spatial features.
- 4. Chain codes or Freeman codes are used to represent the boundary of the image. This is useful for representing the shape as line drawings, planar curves, or contours.
- 5. The idea behind polygonal approximation is to approximate a set of straight line segments to connect the points of interest that are identified by the boundary approximation process.
- 6. The aim of the thinning algorithm is to get a skeleton of an object. Skeleton is a line graph of an object. In many applications, it is necessary to find the skeleton of the object and this is represented by the medial axis transform.

- 7. The total number of pixels present in the boundary approximately gives the length of the object.
- 8. The idea behind the Fourier descriptor is to use FFT to describe the boundary of an object.
- 9. If the convex hull is represented as H, then the set difference S is called the convex deficiency and the concave region can be described using a structure called concavity tree.
- 10. Component labelling is a process of assigning a label to a connected group of pixels.
- 11. Shape features characterize the appearance of the object. Area and perimeter are examples of shape features.
- 12. The main advantage of the topological features is that they are not affected by deformations and are proven to be very useful for recognition purposes.
- 13. Transform features are obtained by scrutinizing the Fourier spectrum in specific regions for image features. High power in small radii corresponds to smooth textures and low power in large radii corresponds to coarse textures.
- 14. Data reduction techniques reduce the original data set while maintaining its integrity. The data can be reduced using some standard techniques such as principal component analysis.