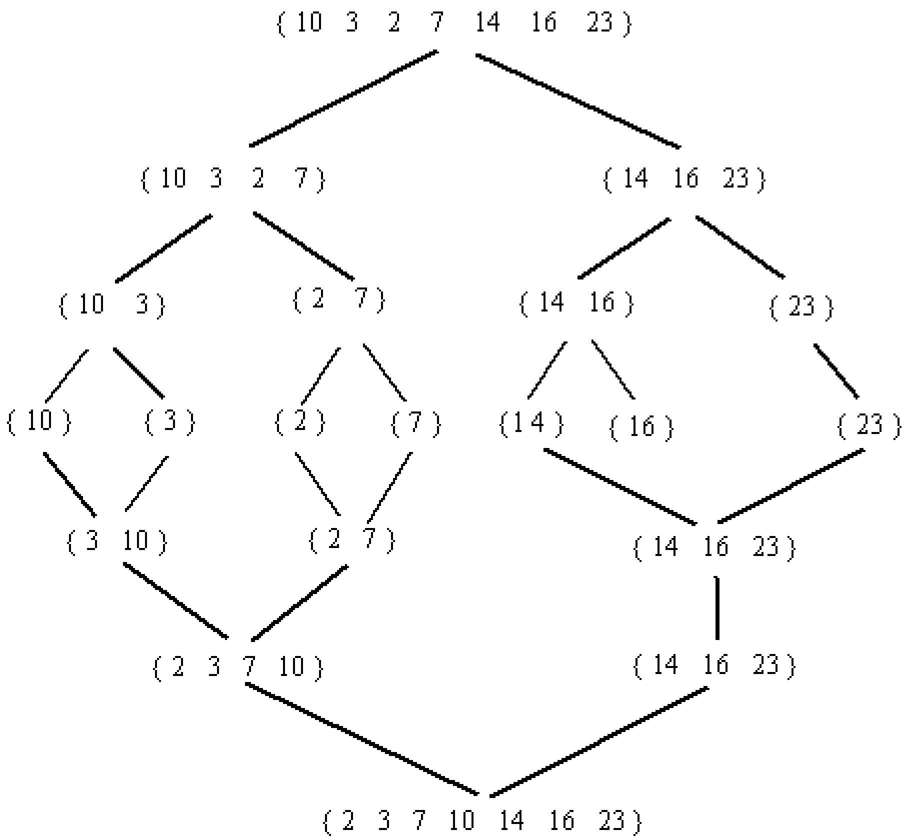
Chapter 8

8.1 Sort the following sequences using a merge sort algorithm in increasing order.
{10, 3, 2, 7, 14, 16, 23}
Solution:



8.2 Sort the following string using a merge sort in increasing order.
a) Polynomial
b) “Test case”

Hint: The characters are treated similar to numbers only. Same as previous problem.

8.3 Use the Hoare method and a Lomuto algorithm for partitioning the following sequences.
a) { 10 15 12 14 17 }
Hint: The intermediate steps of quick sort are shown below:
10, 15,12,14,17
10,1,4,12,15,17,10,1,2,14,15,17

b) { 7 8 12 6 3 }

Hint: The intermediate steps are
7,8,12,6,8
7,3,12,6,8
7,3,6,12,8
6,3,7,12,8
3,6,7,12,8
3,6,7,8,12

8.4 Use Quicksort to sort the sequence
Hint: 3,3,6,4,5,1,10
3,1,6,4,5,3,10
1,3,6,4,5,3,10
1,3,3,4,5,6,10

8.5 Use Quicksort to sort the following sequences of characters in increasing order:

a) Exponential
b) Testcase

Hint: The characters are treated same as the numbers.

8.6 Use the Karatsuba method to carry out the following multiplications

a) 18 X 37

$u = 1\times 10 + 8,$ Here, x = 1 and y=8

$v = 3 \times 10 + 7$, Here, w = 3 and z = 7

Solution: $\mathrm{u} \times \mathrm{v} = \mathrm{xw} \times 10^2 + (yw + xz) \times 10 + yz$

$= 3 \times 10^2 + (24 + 7) \times 10 + 56$

$= 666$

One can verify the result by conventional multiplication.

b) 1864 X 1634

Solution:

$u = 18 \times 10^2 + 64,$ Here, x = 18 and y = 64

$\mathrm{v} = 16 \times 10^2 + 34,$ Here, w = 16 and z = 34

Therefore, $\mathrm{p}_1 = xw = 288$

$\mathrm{p}_2 = yz = 2176$

$\mathrm{p}_3 = [84 \times 50 - 288 - 2176] = 1636$

$uv = 288 \times 10^4 + 1636 \times 10^2 + 2176 = 3045776$

One can verify this by conventional multiplication.

c) 86268 X 172147

Hint: This is same as the previous problem.

8.7 Using Strassen method, multiply the following matrices:

a) $\begin{pmatrix} 1 & 3 \\ 4 & 7 \end{pmatrix} \times \begin{pmatrix} 1 & 2 \\ 7 & 8 \end{pmatrix}$

Here $a_{11}=1$ $a_{21}=4$

$a_{12}=3$ $a_{22}=7$

$b_{11}=1$ $b_{21}=7$

$b_{12}=2$ $b_{22}=8$

$d_1 = (a_{11} + a_{22})(b_{11} + b_{22}) = (1 + 7)(1 + 8) = 8\times9 = 72.$

$d_2 = (a_{21} + a_{22})b_{11} = (4 + 7)(1) = 11.$

$d_3 = a_{11}(b_{12} - b_{22}) = 1(2 - 8) = -6.$

$d_4 = a_{22}(b_{21} - b_{11}) = 7(7 - 1) = 7\times6 = 42.$

$d_5 = (a_{11} + a_{12})b_{22} = (1 + 3)\times8 = 4\times8 = 32.$

$d_6 = (a_{21} – a_{11})(b_{11} + b_{12}) = (4 - 7)(1 + 2) = 3\times3 = 9.$

$d_7 = (a_{12} - a_{22})(b_{21} +b_{22}) = (3 - 7)(7 + 8) = (-4)\times15 = -60.$

$c_{11} = d_1 + d_4 – d_5 +d_7 = 72 + 42 – 32 + (-60)$

$= 22.$

$c_{12} = d_3 + d_5 = -6 + 32 = 26.$

$c_{13} = d_2 + d_4 = 11 + 42 = 53.$

$c_{14} = d_1 + d_3 –d_2 + d_6 = 72 – 6 – 11 + 9 = 64.$

$\therefore \begin{pmatrix} 22 & 26 \\ 53 & 64 \end{pmatrix}$ is the answer

b) $\begin{pmatrix} 1 & 4 & 7 & 8 \\ 4 & 2 & 7 & 6 \\ 1 & 2 & 3 & 4 \\ 7 & 6 & 4 & 7 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$。

Hint: This is similar to the last problem.

8.8 Find the Euclidean distance between the following points.

a) (3,3) and (10,7)

$= \sqrt{(3-10)^2 + (3-7)^2}$

$= \sqrt{49+16} \quad = \sqrt{65} \quad = 8.06$

b) (4,3) and (7,8)

$$= \sqrt{(4-7)^2 + (3-8)^2}$$

$$= \sqrt{9+25} \quad = \sqrt{34} \quad = 5.83$$

8.9 Find the Fourier transforms of the following sequences.

a) { 1, 4, 3, 7 }

Let x = {1,4,3,7}

The kernel y =

1.0000	1.0000	1.0000	1.0000
1.0000	0 - 1.0000i	-1.0000	0 + 1.0000i
1.0000	-1.0000	1.0000	-1.0000
1.0000	0 + 1.0000i	-1.0000	0 - 1.0000i

The Fourier transform is given as

out = y * transpose(x)=

15.0000
-2.0000 + 3.0000i
-7.0000
-2.0000 - 3.0000i

b) { 2, 4 }

Let x = 2,4

The kernel y =

1	1
1	-1

Therefore, The DFT is given as

out = y * transpose(x)

=

6

-2

c) { 8, 6, 1, 4 }

Let x = [8 6 1 4]

The kernel y =

1.0000	1.0000	1.0000	1.0000
1.0000	0 - 1.0000i	-1.0000	0 + 1.0000i
1.0000	-1.0000	1.0000	-1.0000
1.0000	0 + 1.0000i	-1.0000	0 - 1.0000i

The output is given as

19.0000
7.0000 - 2.0000i
-1.0000

7.0000 + 2.0000i

8.10 Prove that the Fourier forward transform and inverse transform are able to recover the original data in the following cases:

a) { 1 4 3 7 }
It can be checked that
DFT result is =
15.0000
-2.0000 + 3.0000i
-7.0000
-2.0000 - 3.0000i from problem 8.9a.
¼ * transpose(kernel) * DFT result gives the original matrix

¼ *	1.0000	1.0000	1.0000	1.0000	* result
	1.0000	0 + 1.0000i	-1.0000	0 - 1.0000i	
	1.0000	-1.0000	1.0000	-1.0000	
	1.0000	0 - 1.0000i	-1.0000	0 + 1.0000i	

= { 1 4 3 7}
Therefore, the inverse is able to recover the original.

b) { 2, 4 }
It can be checked that DFT result is =

6
-2 (from problem 8.9b).

¼ * transpose(kernel) * DFT result gives the original matrix

$$(1/2)* \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} * \text{result}$$
= {2,4}.

Therefore, the inverse is able to recover the original.

c) { 8, 6, 1, 4 }
It can be checked that

DFT result is result =

19.0000

7.0000 - 2.0000i

-1.0000

7.0000 + 2.0000i

¼ * transpose(kernel) * output gives the original matrix

¼ *	1.0000	1.0000	1.0000	1.0000	* result
	1.0000	0 + 1.0000i	-1.0000	0 - 1.0000i	
	1.0000	-1.0000	1.0000	-1.0000	
	1.00	0 - 1.0000i	-1.0000	0 + 1.0000i	

= { 8 6 1 4}

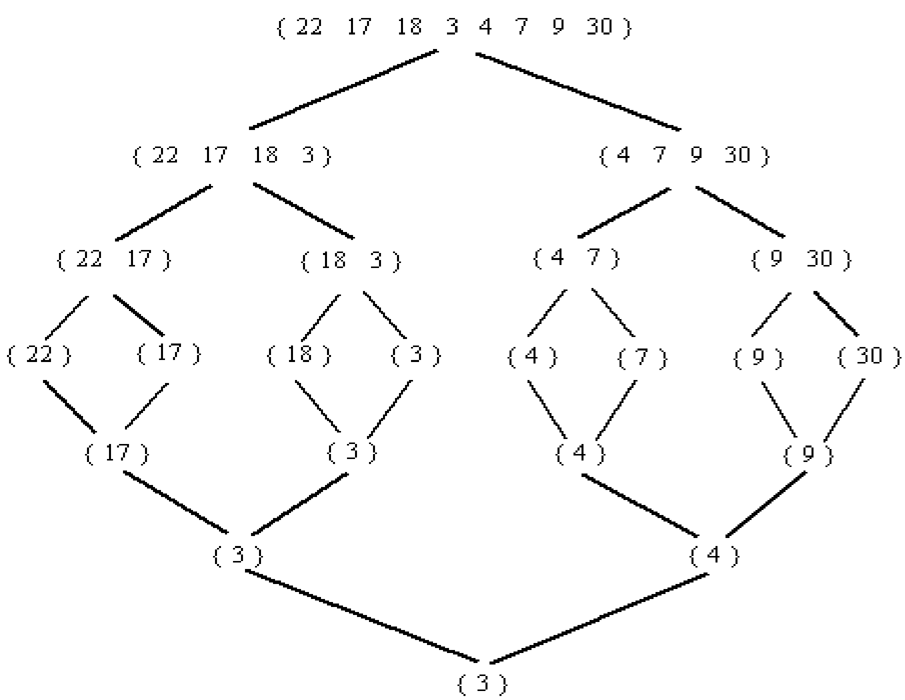
Therefore, the inverse is able to recover the original.

8.11 Using FFT, multiply the following polynomials.

a) $(1 + x + x^2 + x^3) (1 + x + x^2 + x^3)$
The result is equivalent to the convolution of (1 1 1 1) (1 1 1 1)
= 1 2 3 4 3 2 1
These are the coefficients of the resultant polynomial.

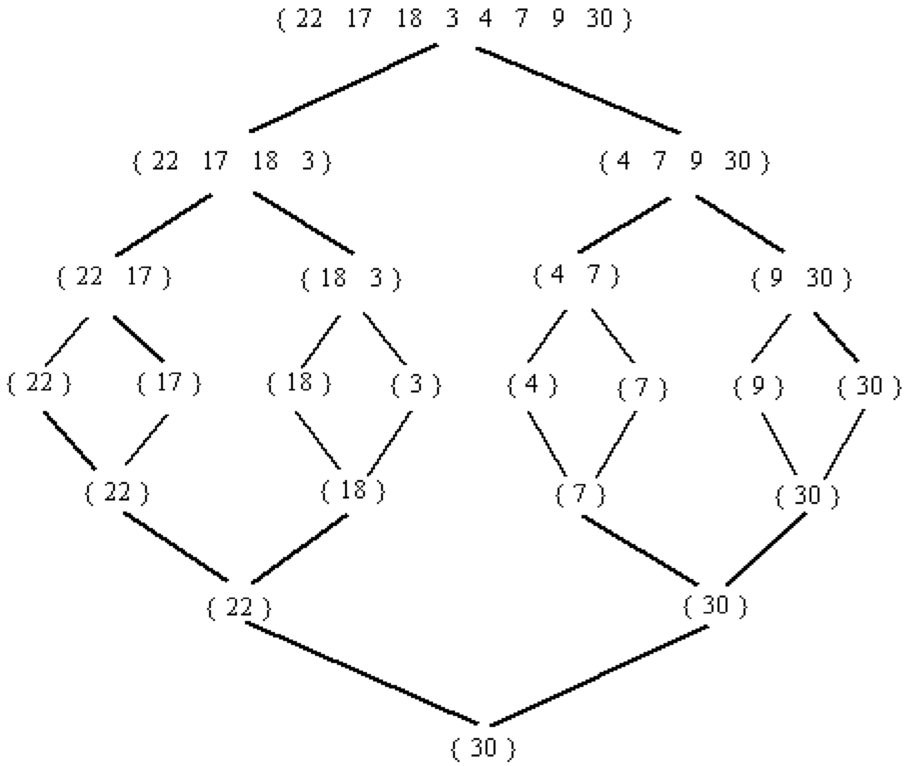
b) $(1 + x + x^2 + 4x^3) (1 + x + x^2 + 2x^3)$
= 1 2 3 8 7 6 8
These are the coefficients of the resultant polynomial.

8.12 Find the maximum and minimum of the array
A = { 22, 17, 18, 3, 4, 7, 9, 30 }



∴ The minimum element is 3.

Similarly the maximum element can be obtained.



∴ The maximum element is 30.