Chapter 19

19.1 Consider the following graph. In addition all the edge costs are given by the following distance adjacency matrix.

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} - & 3 & 4 & 5 & 1 \\ 3 & - & 3 & 2 & 1 \\ 4 & 3 & - & 1 & 2 \\ 5 & 2 & 1 & - & 7 \\ 1 & 1 & 2 & 7 & - \end{pmatrix} \end{array}$$

Apply nearest neighbour heuristics for this graph and obtain TSP tour.

Solution:

Let 1 be the starting vertex. The nearest vertex is 5. So the path is 1–5. From 5 nearest is 2.

∴ The path is 1–5–2. From 2, the nearest is 4. So the path is 1–5–2–4. From 4, nearest is 1.

∴ The path is 1–5–2–4–3. From 3, one can return to the node 1.

19.2 Use Fermat primality testing and check whether the following numbers are prime or not.

a) 17

Let p be 17

i.e., $2^{17-1} \bmod 17$ let a = 2

$2^{16} \bmod 17 \equiv 1$ $\boxed{2^{16} = 65 \times 36}$

∴ 17 is a prime number.

b) 227

Let p be 227

i.e., $2^{227-1} \bmod 227 \equiv 1$

∴ 227 is a prime number.

19.3 Use Fermat testing for Carmichael number like 561 and prove that the algorithm will not work.

Hint: $561 = 3 \cdot 11 \cdot 17$

19.4 Consider the following graph.

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$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{pmatrix} - & 3 & 4 & 5 & 1 & 2 \\ 3 & - & 3 & 2 & 1 & 3 \\ 4 & 3 & - & 1 & 2 & 4 \\ 5 & 2 & 1 & - & 7 & 1 \\ 1 & 1 & 2 & 7 & - & 5 \\ 2 & 3 & 4 & 1 & 5 & - \end{pmatrix} \end{array}$$

Apply the nearest neighbour and obtain TSP hour.

***Solution*:**

Assume 1 is starting vertex

∴ The nearest node is 5.

1–5

From 5, nearest node is 2.

1–5–2

From 2, nearest node is 4

1–5–2–4

From 4, nearest node is 3

1–5–2–4–3

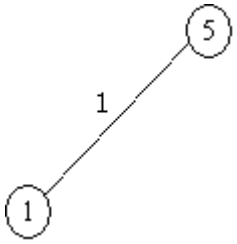
From 3, the unvisited node is 6.

∴ the tour is 1–5–2–4–3–6–1.

19.5 Use the above graph and apply multi-fragment Heuristics.

***Solution*:**

The minimum edge is 1–5



The minimum edge is 5–2

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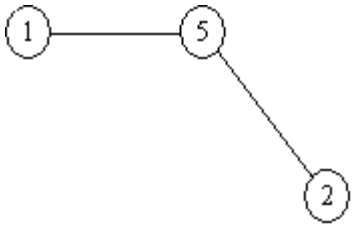
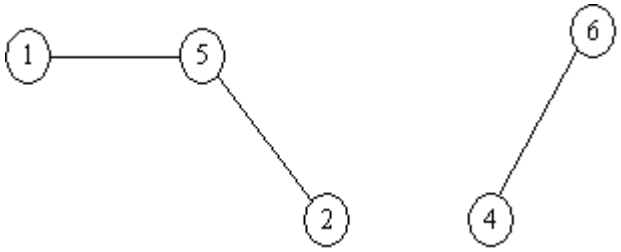
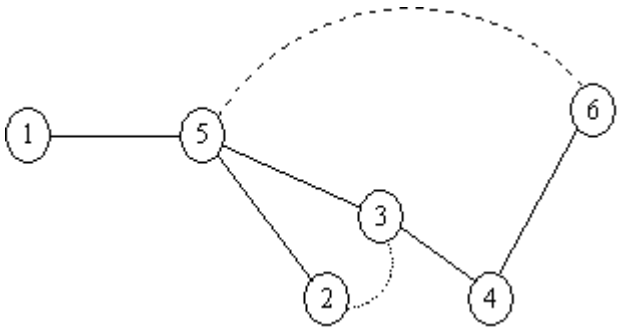


The minimum edge is 6–4



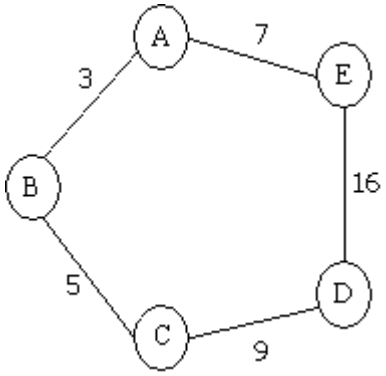
The minimum edge is 4–3



The path can be constructed by adding a cross edge.

1–5–2–3–4–6–1.

19.6 Consider the following graph



***Solution*:**

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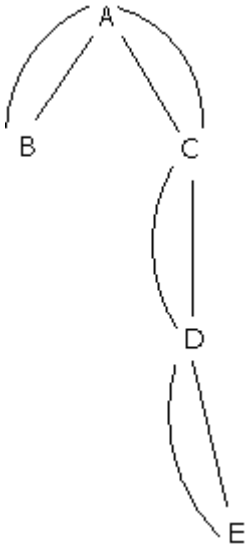
Construct MST as follows:



The final tour is

A—B—A—C—D—E—D—C—A

Remove the duplicate nodes to get TSP tour.

19.7 Consider the above graph and apply the nearest neighbour heuristic and obtain TSP tour.

Solution:

Assume that the starting node is A.

∴ The nearest node is B

So the path is A⟶B

∴ Then, the path can be extended to E

A⟶B⟶E

∴ Then the path can be extended to D

The path is A⟶B⟶E⟶D

∴ the final path is A⟶B⟶E⟶D⟶A

19.8 Find the vertex cover for the following graph.

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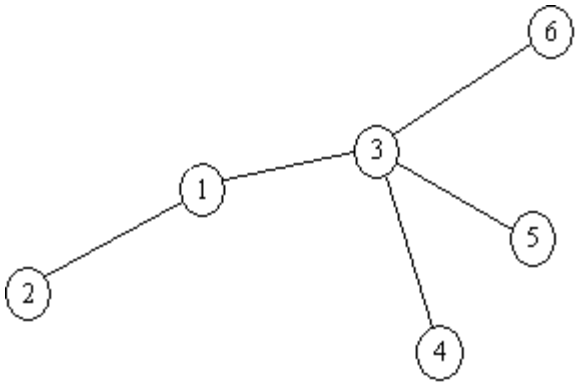

***Solution*:**

The answer is {3, 1 }

19.9 Find the set cover of the following problem. Let there be four laboratory procedures 1, 2, 3 and 4 and four lab technicians L_1, L_2, L_3 and L_4 . The technician who know how to construct the procedure are as follows:

L_1, : c_1,c_3

L_2 : c_2,c_3,c_4

L_3 : c_4

L_4 : c_3

Find the set cover.

Solution:

The answer is { L_1, L_2 }

19.10 For the following knapsack. Apply a 2-Approxmate algorithm and show intermediate results. w = 10.

Item	w_i	p_i
1	2	8
2	3	15
3	4	16
4	3	21

***Solution*:**

$$\frac{p_1}{w_1} = \frac{8}{2} = 4$$

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$$\frac{p_2}{w_2} = \frac{15}{3} = 5$$

$$\frac{p_3}{w_3} = \frac{16}{4} = 4$$

$$\frac{p_4}{w_4} = \frac{21}{3} = 7$$

∴ Sort it as $\begin{matrix} p_1 & p_2 & p_3 & p_4 \\ \{ 7, & 5, & 4, & 4 \} \end{matrix}$

Add item 4 The $w = 10 - 3 = 7$

Add item $p_2 = 3$ $w = 7 - 3 = 4$

Add item $p_3 = 2$ $w = 4 - 4 = 0$

∴ The items that can be added are { 4, 3, 2 }.

Item 1 can't be added as the weight crossing capacity.

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