Chapter 15

15.1 Write a LC procedure for the 15 puzzle problem.

Solution:

Algorithm branch-bound-puzzle (page no.551) can be used to solve 15-puzzle as well.

15.2 Write a procedure for the assignment problem.

Solution:

1. Enqueue 'Start' node to priority queue priority queue p_a.
2. Generate children.
3. Enqueue the children nodes to p_a.
4. If the goal node is obtained, then all assignments are made.
   Report success else go to step2.
5. End

15.3 Write a Branch and Bound procedure for the Knapsack problem.

***Solution*:**

1. Arrange the items such that
2. Construct the binary tree.
3. Compute the lower bound.
4. End

15.4 Write a Brunch and Bound procedure for TSP

***Solution*:**

Step 1 : Let C be the reduced cost matrix.

Step 2 : Set $C(1,1)=\infty$.

Step 3 : Reduce the distance matrix for the rows and columns that contains ∞.

Step 4 : Compute the lower bound.

Step 5 : End.

15.5 Consider the following 8 puzzle game and draw and illustrate a Branch and Bound technique.

$$\begin{matrix} 1 & 2 & 3 \\ 4 & - & 5 \\ 6 & 7 & 8 \end{matrix} \quad \rightarrow \quad \begin{matrix} 1 & 2 & 3 \\ 7 & - & 5 \\ 4 & 6 & 8 \end{matrix}$$

***Solution*:**

$$\begin{array}{ccc} 1 & 2 & 3 \\ 4 & - & 5 \\ 6 & 7 & 8 \end{array}$$

$$\downarrow$$

$$\begin{array}{ccc} 1 & 2 & 3 \\ 4 & 7 & 5 \\ 6 & - & 8 \end{array}$$

$$\downarrow$$

$$\begin{array}{ccc} 1 & 2 & 3 \\ 4 & 7 & 5 \\ - & 6 & 8 \end{array}$$

$$\downarrow$$

$$\begin{array}{ccc} 1 & 2 & 3 \\ - & 7 & 5 \\ 4 & 6 & 8 \end{array}$$

$$\downarrow$$

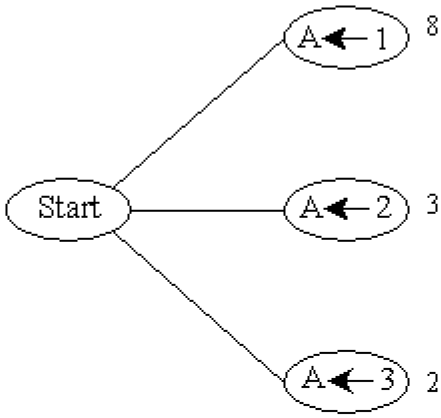
$$\begin{array}{ccc} 1 & 2 & 3 \\ 7 & - & 5 \\ 4 & 6 & 8 \end{array}$$

The path to the goal is a shown below.

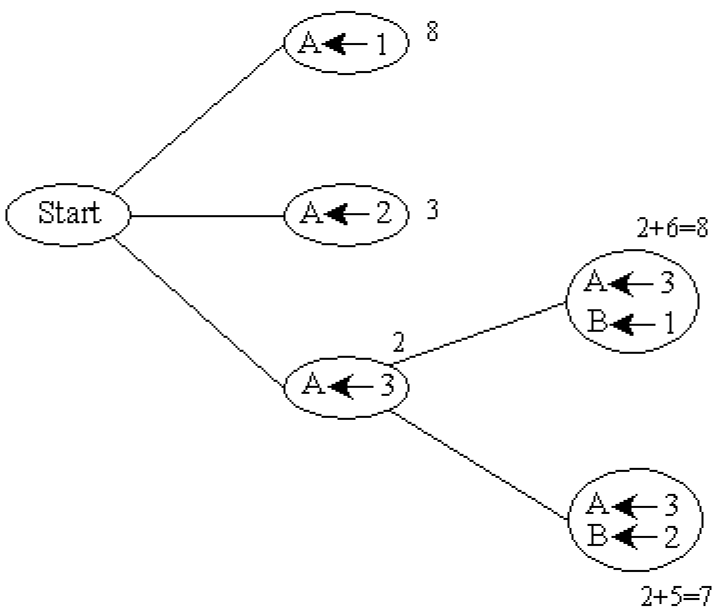
15.6 Solve the following assignment problem using the cost matrix for assigning tasks to workers.

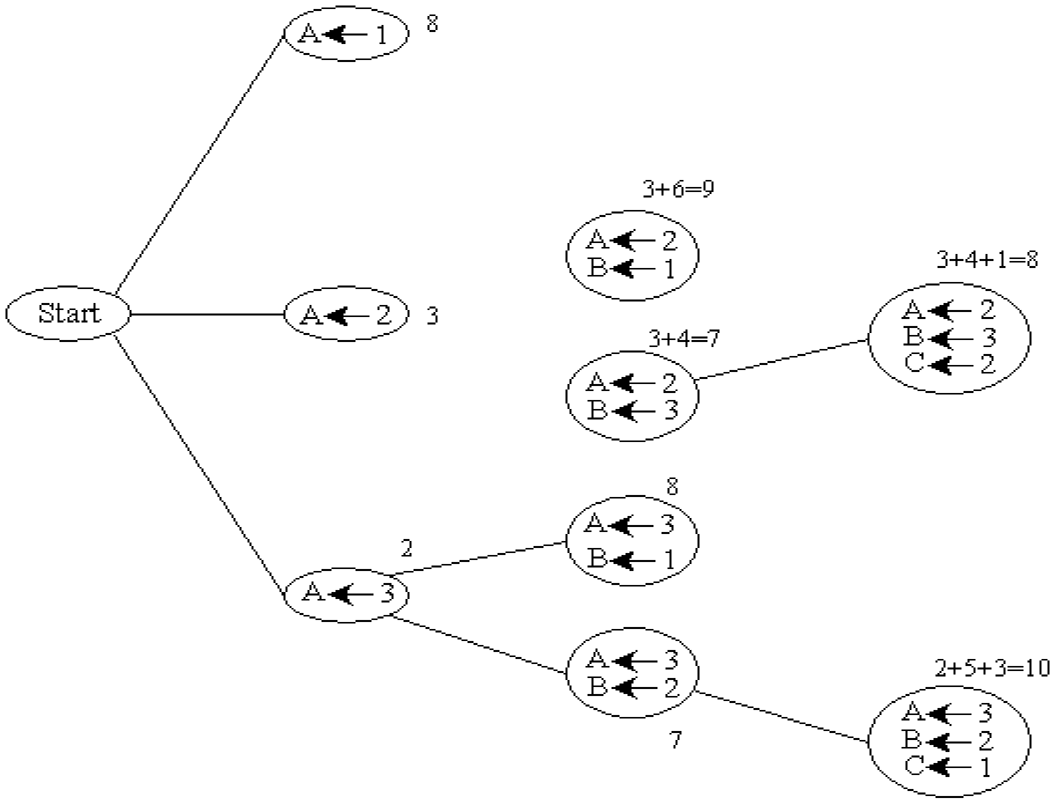
W \ T	1	2	3
A	8	3	2
B	6	5	4
C	3	1	2

Solution:



Compute LB as 8+5+2 = 15





Therefore assign the tasks as follows:

A←2

B←3

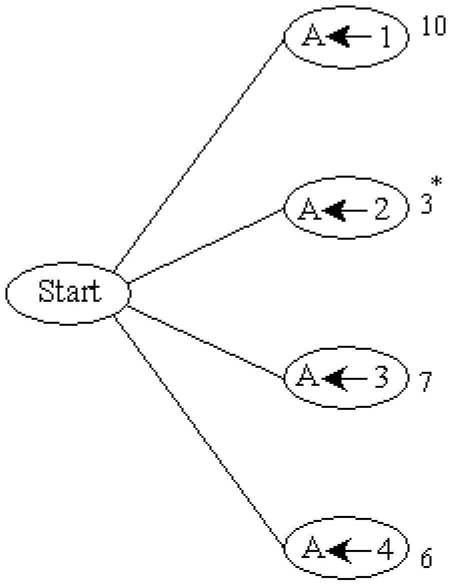
C←2 with cost of 8.

15.7 Use the following cost matrix and solve the assignment problem.

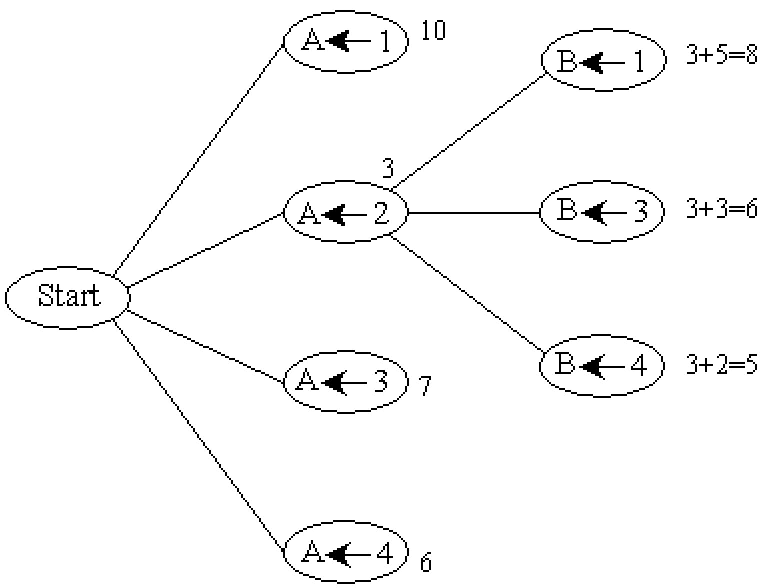
Workers \ Tasks	1	2	3	4
A	10	3	7	6
B	5	4	3	2
C	1	3	6	7
D	8	9	5	3

***Solution*:**

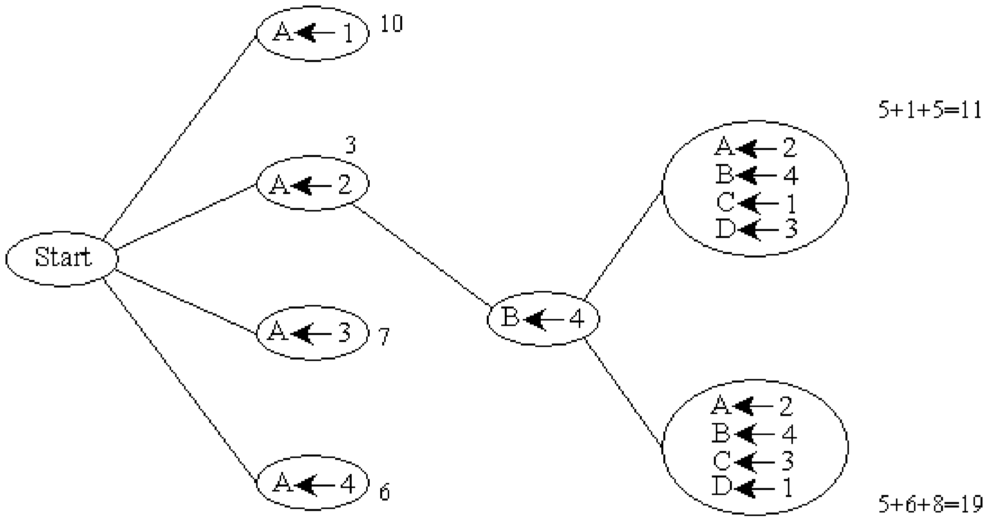
The LB is 10 + 4 + 6 + 3 = 23



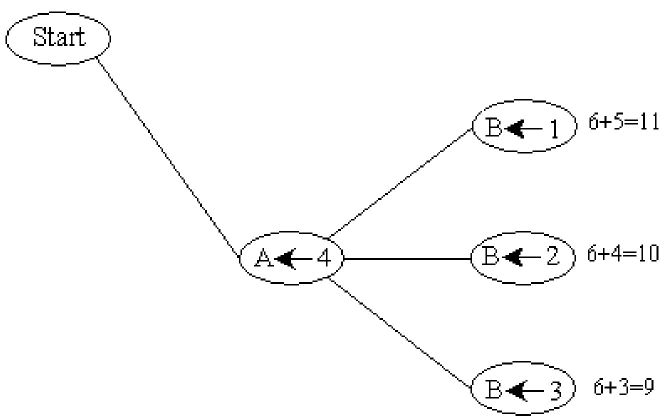
Expand the node further



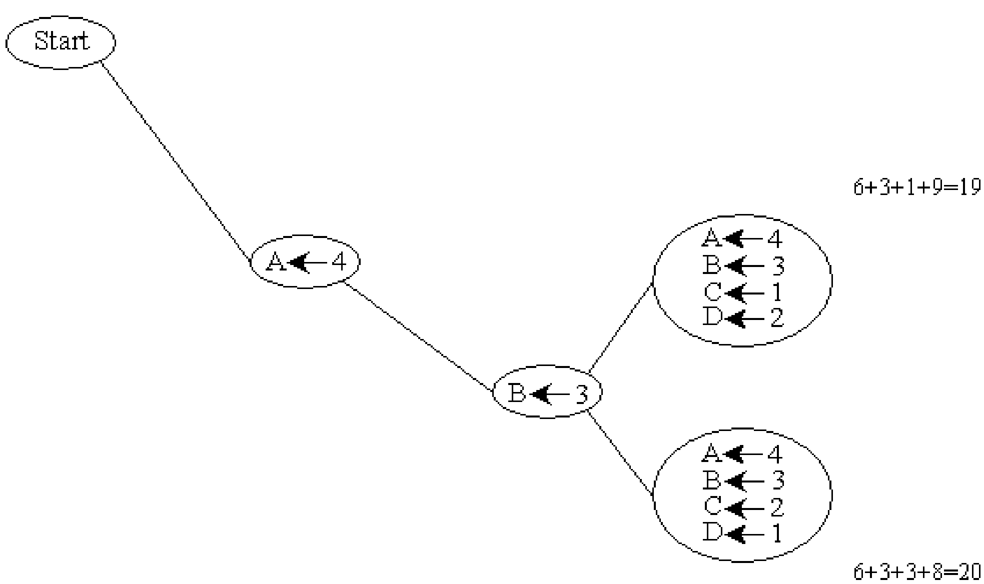
Minimum is 5. So expand it further.



Expand A←4 further



Minimum is 9 and expand further



The optimal assignment is

A←2, B←4, C←1, D←3

with minimum cost of 11.

15.8 Solve the following Knapsack problem using the Branch–and–Bound technique. Assume $w = 12$.

Items	w_i	p_i
1	2	16
2	3	20
3	4	24

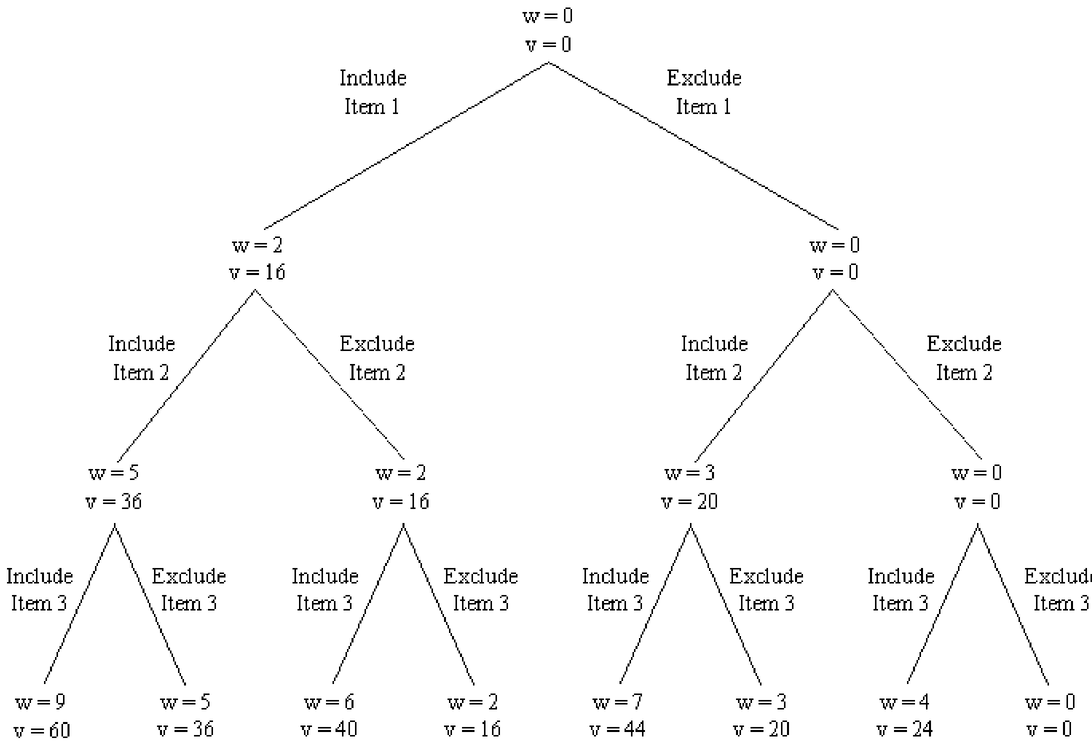
***Solution*:**

$$\frac{v_1}{w_1} = \frac{16}{2} = 8$$

$$\frac{v_2}{w_2} = \frac{20}{3} = 6.6$$

$$\frac{v_3}{w_3} = \frac{24}{4} = 6$$

$\therefore$ The items are already in sorted order.



∴ The goal is $w = 9$ with profit of 60.

15.9 Solve the following Knapsack problem using the Branch–and–Bound technique. Assume Knapsack capacity $w = 12$.

Items	w_i	p_i
1	2	10
2	3	12
3	4	20
4	5	25

Solution:

$$\frac{v_1}{w_1} = \frac{10}{2} = 5$$

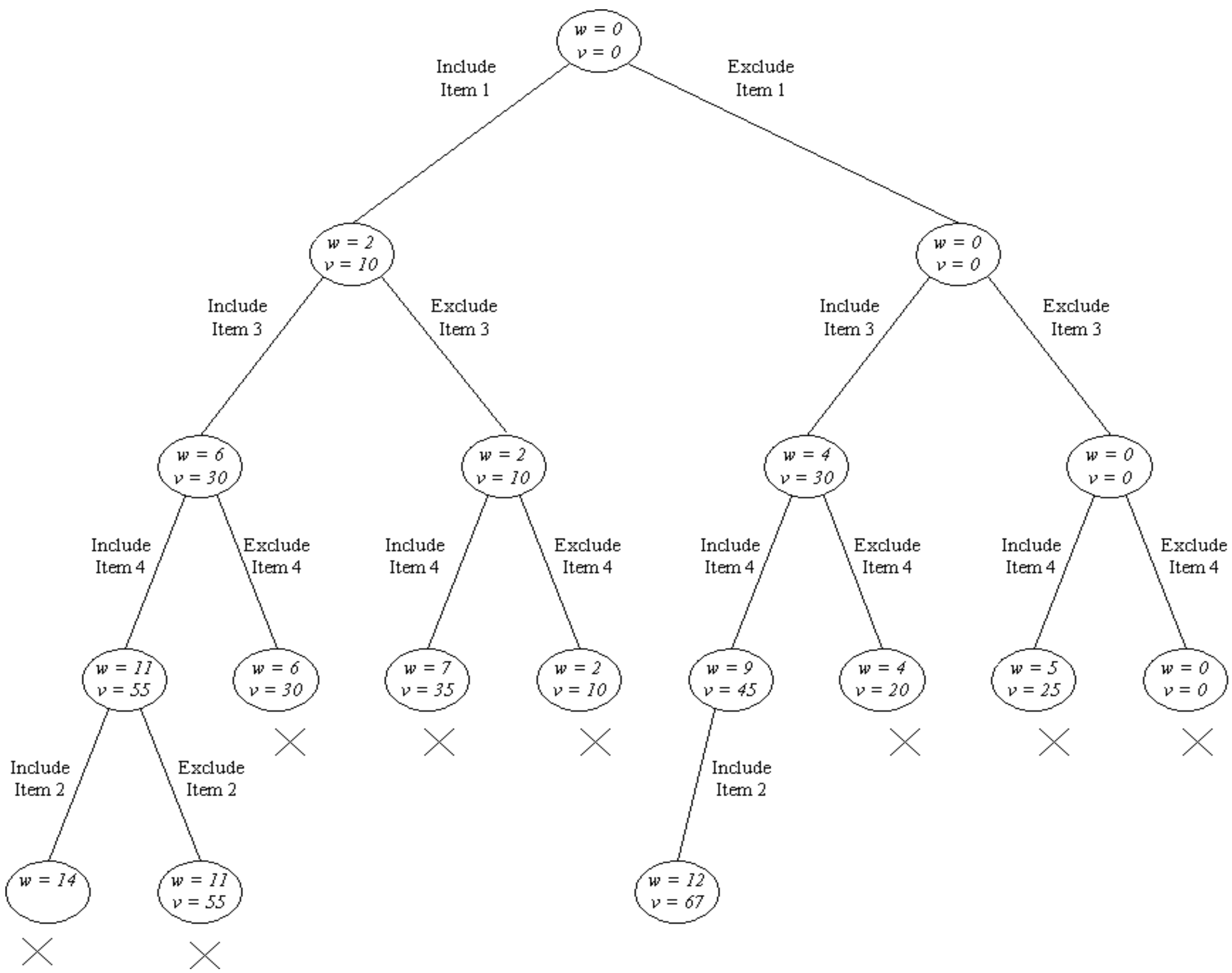
$$\frac{v_2}{w_2} = \frac{12}{3} = 4$$

$$\frac{v_3}{w_3} = \frac{20}{4} = 5$$

$$\frac{v_4}{w_4} = \frac{25}{5} = 5$$

$\therefore$ Arrange items and rearrange it to get as follows:

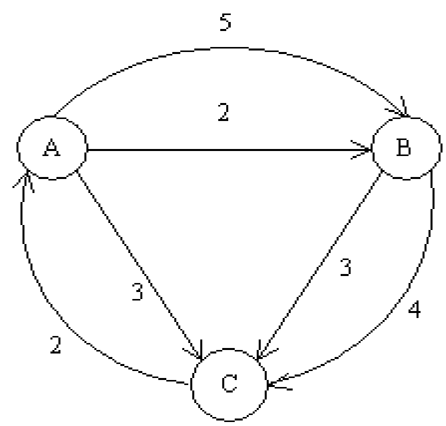
Items	w_i	p_i
1	2	10
3	4	20
4	5	25
2	3	12



$\therefore$ It can be observed that the best combination is Include item 3, 4 and 2 with profit 67.

15.10 Solve the following TSP using Branch–and–Bound technique.

Items	*A*	*B*	*C*
A	∞	2	3
B	5	∞	3
C	2	4	∞



$$\underset{\textit{row real}}{\begin{pmatrix} \infty & 2 & 3 \\ 5 & \infty & 3 \\ 2 & 4 & \infty \end{pmatrix}\begin{matrix} 2 \\ 3 \\ 2 \end{matrix}} \rightarrow \underset{\textit{row reduction}}{\begin{pmatrix} \infty & 0 & 1 \\ 2 & \infty & 0 \\ 0 & 2 & \infty \end{pmatrix}}$$

Total reduction is 2 + 3 + 2 = 7. This is the lower bound.

$\boxed{\text{Path (A,B)}}$

Set 1^{st} row and 1^{st} column ∞ ∞ and Set A(2,1) as ∞. This gives.

$$\begin{pmatrix} \infty & \infty & \infty \\ \infty & \infty & 0 \\ 0 & \infty & \infty \end{pmatrix}$$

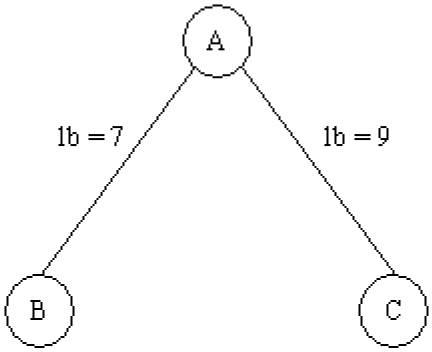
This gives reduction as 7 + 0 = 7

$\boxed{\text{Path (A,C)}}$

$$\begin{pmatrix} \infty & 0 & \infty \\ 2 & 0 & \infty \\ \infty & 2 & \infty \end{pmatrix}$$

This gives reduction 7 + 2 = 9

So the tree would be



So the vertex B is selected. Only the left out node is C.

∴ The TSP tour is $A \to B \to C \to A$.

15.11 Solve the following TSP using Hungarian technique.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} \infty & 2 & 3 & 4 \\ 1 & \infty & 4 & 3 \\ 2 & 3 & \infty & 4 \\ 4 & 3 & 2 & \infty \end{pmatrix} \end{array}$$

Solution:

Perform row reduction as follows.

$$\begin{pmatrix} \infty & 2 & 3 & 4 \\ 1 & \infty & 4 & 3 \\ 2 & 3 & \infty & 4 \\ 4 & 3 & 2 & \infty \end{pmatrix} \begin{array}{c} 2 \\ 1 \\ 2 \\ 2 \end{array}$$

To get

$$\begin{pmatrix} \infty & 0 & 1 & 2 \\ 0 & \infty & 3 & 2 \\ 0 & 1 & \infty & 2 \\ 2 & 1 & 0 & \infty \end{pmatrix}$$

Perform column reduction to get

$$\begin{array}{c}\begin{pmatrix}\infty & 0 & 1 & 2\\ 0 & \infty & 3 & 2\\ 0 & 1 & \infty & 2\\ 2 & 1 & 0 & \infty\end{pmatrix}\\ \hline \begin{matrix}0 & 0 & 0 & 2\end{matrix}\end{array} \Rightarrow \begin{pmatrix}\infty & 0 & 1 & 0\\ 0 & \infty & 3 & 0\\ 0 & 1 & \infty & 0\\ 2 & 1 & 0 & \infty\end{pmatrix}$$

One can choose ‘O’ to prick the next vertex. By carefully selecting and excluding that row and column we get the path as

$$A \longrightarrow B \longrightarrow D \longrightarrow C \longrightarrow A$$

This is with the cost 9.